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Introduction to Supersymmetry (at Colliders)

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OUTLINE

1. **Introduction: Supersymmetry, what and why?**

2. **Formal basics: 2-Component Fermions**

3. **Motivation for Supersymmetry**

4. **MSSM**

5. **R-parity Conservation vs R-Parity Violation, or P_6 vs B_3**

6. **Signals at the LHC**

References

Books:

- Weak scale supersymmetry: From superfields to scattering events,
  H. Baer & X. Tata

- Theory and phenomenology of sparticles: An account of 4-dim N=1 supersymmetry in high energy physics,
  M. Drees, R. Godbole, P. Roy

- Supersymmetry in Particle Physics. An Elementary Introduction,
  I.J.R. Aitchison

Review Papers

- Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry,
  HD, H. E. Haber, S. .P. Martin;
  e-Print: arXiv:0812.1594

- A Supersymmetry Primer,
  S. P. Martin,
  hep-ph/9709356

- The Search for Supersymmetry: Probing Physics Beyond the Standard Model
  H. .E. Haber, G. .L. Kane,
  Phys. Rept. 117 (1985) 75.

- Supersymmetry and the MSSM: An Elementary introduction,
  Ian J.R. Aitchison,
  hep-ph/0505105

SUPERSYMMETRY

- **BASIC IDEA:** extend symmetry transformations of the Standard Model

$$|\mathbf{FERMION}\rangle \overset{Q}{\longleftrightarrow} |\mathbf{BOSON}\rangle$$

- **INFINITESIMAL FIELD TRANSFORMATION:**

SCALAR: $\phi \longrightarrow \phi' = \phi + \Delta\phi;$ $\qquad \Delta\phi = \epsilon \cdot \psi$

FERMION: $\psi \longrightarrow \psi' = \psi + \Delta\psi;$ $\qquad \Delta\psi = \epsilon \cdot \phi$

- The quantum fields: ϕ, ψ form a pair, a supermultiplet

- ϵ is a constant spinor in global supersymmetry.

MOTIVATION

SO FAR NO (DIRECT) EXPERIMENTAL EVIDENCE FOR SUSY

Why would you do such a thing?

A Priori:

1. Maximal External Symmetries in Nature
2. Solution to the Hierarchy Problem: $M_W \longleftrightarrow M_{\text{Planck}}$

A Posteriori:

1. Unification of Standard Model gauge interactions at $M_X = \mathcal{O}(10^{16}\,\text{GeV})$
2. Dark matter (R-parity) / Neutrino masses (Baryon Triality)
3. Falsifiable at the LHC

SUPERFIELDS

- Combine electron $\psi_e(s = \frac{1}{2})$ and selectron $\phi_{\tilde{e}}(s = 0)$ to chiral superfield

$$\Phi_e \sim \phi_{\tilde{e}_L} + \theta \psi_{e_L} = E_L,$$

- Similarly for other fermion fields:

$$L = \begin{pmatrix} N \\ E \end{pmatrix}_L \sim \begin{pmatrix} \phi_{\tilde{\nu}} + \theta \psi_\nu \\ \phi_{\tilde{e}} + \theta \psi_e \end{pmatrix}_L, \quad E^c \sim \phi^*_{\tilde{e}} + \theta \psi^c_{e_R}$$

$$Q = \begin{pmatrix} U \\ D \end{pmatrix}_L \sim \begin{pmatrix} \phi_{\tilde{u}} + \theta \psi_u \\ \phi_{\tilde{d}} + \theta \psi_d \end{pmatrix}_L, \quad U^c \sim \phi^*_{\tilde{u}} + \theta \psi^c_{u_R}, \quad D^c \sim \phi^*_{\tilde{d}} + \theta \psi^c_{d_R}$$

- Plus Higgs superfields: $H_1,\ H_2$

- Gauge bosons form vector supermultiplets: $[\lambda_{\tilde{\gamma}}(s = \frac{1}{2}),\ A^\gamma_\mu(s = 1)]$

- Thus we must double field content

SUSY SPECTRUM

Standard Model + SUSY $\Longrightarrow$ Double Spectrum (+2 Higgs Doublets)

e^- (spin $= \frac{1}{2}$)	$\longleftrightarrow$	$\tilde{e}$ $(s = 0)$	scalar electron
top t $(s = \frac{1}{2})$	$\longleftrightarrow$	$\tilde{t}$ $(s = 0)$	scalar top
$H^{\pm}$ $(s = 0)$	$\longleftrightarrow$	$\tilde{H}^{\pm}$ $(s = \frac{1}{2})$	Higgsino
H^0, h^0 $(s = 0)$	$\longleftrightarrow$	$\tilde{H}^0$, $\tilde{h}^0$ $(s = \frac{1}{2})$	Higgsino
$W^{\pm}$ $(s = 1)$	$\longleftrightarrow$	$\tilde{W}^{\pm}$ $(s = \frac{1}{2})$	Wino
B, W^0 $(s = 1)$	$\longleftrightarrow$	$\tilde{B}$, $\tilde{W}^0$ $(s = \frac{1}{2})$	Bino, neutral Wino
$g_{a=1,\ldots,8}$ $(s = 1)$	$\longleftrightarrow$	$\tilde{g}_a$ $(s = \frac{1}{2})$	Gluino

- No supermultiplet has yet been observed – why not?

SUPERSYMMETRY BREAKING

- Supersymmetry: $\mathrm{Mass}(e^-) = \mathrm{Mass}(\tilde{e}^-_{L,R})$
- Must be broken: spontaneously in local supersymmetry
- Must preserve solution to hierarchy problem: new mass terms in $\mathscr{L}$

$$\mathrm{Mass}(\tilde{e}^-_L) \longrightarrow \quad m^2_{\tilde{e}_L} \phi^\dagger_{\tilde{e}_L} \phi_{\tilde{e}_L} \ \in \ \mathscr{L}$$

- Similarly for other scalar partners: $m^2_{\tilde{e}_R}$, $m^2_{\tilde{q}_L}$, $m^2_{\tilde{q}_R}$
- Hierarchy problem: $m_{\tilde{e}} = \mathcal{O}(1\,\mathrm{TeV})$
- Testable at the LHC

INTRODUCTION II

- Dirac Spinor in chiral representation: $\Psi(x) = \begin{pmatrix} (\psi_L)_\alpha(x) \\ (\psi_R)^{\dot\alpha}(x) \end{pmatrix}$

- $(\psi_L)_\alpha$ and $(\psi_R)^{\dot\alpha}$, 2-comp. spinors, $\alpha,\, \dot\alpha = 1, 2$

- In the Standard Model, transform differently under gauge transf.

- ψ_R is an SU(2) singlet; ψ_L is part of an SU(2) doublet

$$\psi_L^{a\prime}(x) = \mathrm{exp}[i\alpha_j(x)\mathbf{T}^j]^a_b\, \psi_L^b(x),$$

- $T^j = \frac{1}{2}\tau^j$, $a, b = 1, 2$ (fund. rep.), $j = 1, 2, 3$ (adjoint) gauge indices

- Internal symmetry: SU(2) generators and Lorentz generators commute

Lorentz Algebra

- How does a field transform under Lorentz transformations?

$$\Phi'_\alpha(x') = M_R(\Lambda)\Phi(x) = \exp\left(-\tfrac{i}{2}\theta_{\mu\nu}J^{\mu\nu}\right)_\alpha^{\ \beta}\,\Phi_\beta(x)\,,$$

- Note: $x'^\mu = \Lambda^\mu{}_\nu x^\nu$
- Generators $J^{\mu\nu}$ satisfy commutation relations

$$[J^{\mu\nu}\,,\,J^{\lambda\kappa}] = i(g^{\mu\kappa}\,J^{\nu\lambda} + g^{\nu\lambda}\,J^{\mu\kappa} - g^{\mu\lambda}\,J^{\nu\kappa} - g^{\nu\kappa}\,J^{\mu\lambda})$$

- Redefine the combinations

$$J^i \equiv \tfrac{1}{2}\epsilon^{ijk}J_{jk}\,, \qquad K^i \equiv J^{0i}\,, \qquad i,j,k = 1,2,3$$

- J^i generate 3-dim rot. in space
- K^i generate the Lorentz-boosts of the fields
- Infinitesimal: $M_R(\Lambda) = \exp(-\frac{i}{2}\theta_{\mu\nu}J^{\mu\nu}) \simeq I - i\vec{\boldsymbol{\theta}}\cdot\vec{\boldsymbol{J}} - i\vec{\boldsymbol{\zeta}}\cdot\vec{\boldsymbol{K}}\,,$

- J^i and K^i satisfy the commutation relations

$$\begin{aligned} [J^i, J^j] &= i\epsilon^{ijk} J^k\,, \\ [J^i, K^j] &= i\epsilon^{ijk} K^k\,, \\ [K^i, K^j] &= -i\epsilon^{ijk} J^k\,. \end{aligned}$$

- As expected, the J^i satisfy the angular momentum algebra

- Now redefine: $\vec{\boldsymbol{S}}_+ \equiv \frac{1}{2}(\vec{\boldsymbol{J}} + i\vec{\boldsymbol{K}})\,,\qquad \vec{\boldsymbol{S}}_- \equiv \frac{1}{2}(\vec{\boldsymbol{J}} - i\vec{\boldsymbol{K}})\,.$

- Commutation relations decouple into two independent SU(2) algebras

$$\begin{aligned} [S^i_+, S^j_+] &= i\epsilon^{ijk} S^k_+\,, \\ [S^i_-, S^j_-] &= i\epsilon^{ijk} S^k_-\,, \\ [S^i_+, S^j_-] &= 0\,. \end{aligned}$$

- Two independent SU(2), “angular momenta” quantum numbers:

$$\ell_1 = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots \qquad\qquad \ell_2 = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots$$

Classification of Fields

- Use this to classify representations of Lorentz algebra: (ℓ_1, ℓ_2)
- Scalar field: $(\ell_1 = 0, \ell_2 = 0) \quad \longrightarrow \phi(x)$:
- Left-chiral fermion fields: $(\frac{1}{2}, 0), \quad \longrightarrow \chi_\alpha(x)$

$$\Longrightarrow \quad \vec{\boldsymbol{J}} = \vec{\boldsymbol{\sigma}}/2 \ \text{ and } \ \vec{\boldsymbol{K}} = -i\vec{\boldsymbol{\sigma}}/2$$

$$\Longrightarrow \quad M_{(\frac{1}{2},0)}(\Lambda) \simeq I - i\vec{\boldsymbol{\theta}}\cdot\vec{\boldsymbol{\sigma}}/2 - \vec{\boldsymbol{\zeta}}\cdot\vec{\boldsymbol{\sigma}}/2$$

- The $(\frac{1}{2}, 0)$ fermion fields, $\chi_\alpha(x)$, transform under this matrix

- Right-chiral fermion fields: $(0, \frac{1}{2})$, $\longrightarrow \eta^{\dagger\dot{\alpha}}(x)$

$$\Longrightarrow \quad \vec{\boldsymbol{J}} = \vec{\boldsymbol{\sigma}}/2 \text{ and } \vec{\boldsymbol{K}} = +i\vec{\boldsymbol{\sigma}}/2$$

$$\Longrightarrow \quad M_{(0,\frac{1}{2})}(\Lambda) \simeq I - i\vec{\boldsymbol{\theta}}\cdot\vec{\boldsymbol{\sigma}}/2 + \vec{\boldsymbol{\zeta}}\cdot\vec{\boldsymbol{\sigma}}/2 = (M^{-1}_{(\frac{1}{2},0)}(\Lambda))^{\dagger}$$

- The $(\frac{1}{2}, 0)$ fermion fields, $\eta^{\dagger\dot{\alpha}}(x)$, transform under this different matrix

- Complex conjugate; thus $\eta^{\dagger}_{\dot{\alpha}} = (\eta_{\alpha})^{\dagger}$

Two-Component Fermion Field Theory

- Dirac fermion: $$\Psi_D(x) = \begin{pmatrix} \chi_\alpha(x) \\ \eta^{\dot{\alpha}\dagger}(x) \end{pmatrix}$$

- Majorana fermion: $$\Psi_M(x) = \begin{pmatrix} \chi_\alpha(x) \\ \chi^{\dot{\alpha}\dagger}(x) \end{pmatrix}$$

- Can now construct entire fermionic field theory out of two-component fermions

$$\mathscr{L} = i\xi^\dagger \overline{\sigma}^\mu \partial_\mu \xi - \tfrac{1}{2} m(\xi\xi + \xi^\dagger \xi^\dagger)$$

$$\mathscr{L} = i\chi^\dagger \overline{\sigma}^\mu \partial_\mu \chi + i\eta^\dagger \overline{\sigma}^\mu \partial_\mu \eta - m(\chi\eta + \chi^\dagger \eta^\dagger)$$

- $\sigma^\mu = (1_{2\times 2}\,; \vec{\boldsymbol{\sigma}})\,, \qquad \overline{\sigma}^\mu = (1_{2\times 2}\,; -\vec{\boldsymbol{\sigma}})$

Quantized Fields

$$\xi_\alpha(x) = \sum_s \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^{3/2}(2E_{\boldsymbol{p}})^{1/2}} \left[x_\alpha(\vec{\boldsymbol{p}}, s)a(\vec{\boldsymbol{p}}, s)e^{-ip\cdot x} + y_\alpha(\vec{\boldsymbol{p}}, s)a^\dagger(\vec{\boldsymbol{p}}, s)e^{ip\cdot x}\right]$$

$$\xi^\dagger_{\dot\alpha}(x) = \sum_s \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^{3/2}(2E_{\boldsymbol{p}})^{1/2}} \left[x^\dagger_{\dot\alpha}(\vec{\boldsymbol{p}}, s)a^\dagger(\vec{\boldsymbol{p}}, s)e^{\,ip\cdot x} + y^\dagger_{\dot\alpha}(\vec{\boldsymbol{p}}, s)a(\vec{\boldsymbol{p}}, s)e^{-ip\cdot x}\right]$$

- Two-component spinor wave functions

$$x_\alpha(\vec{p}, s), \quad y_\alpha(\vec{p}, s), \quad x^\dagger_{\dot\alpha}(\vec{p}, s), \quad y^\dagger_{\dot\alpha}(\vec{p}, s)$$

- Convenient formalism for supersymmetric calculations
- See HD, Haber, Martin: arXiv:0812.1594 (300+ pages) for details

Poincaré Algebra

- Including generator of translations, P^μ

$$
\begin{aligned}
[P^\mu, P^\nu] &= 0\,, \\
\left[J^{\mu\nu}, P^\lambda\right] &= i(g^{\nu\lambda}P^\mu - g^{\mu\lambda}P^\nu)\,, \\
\left[J^{\alpha\beta}, J^{\rho\sigma}\right] &= i(g^{\beta\rho}\,J^{\alpha\sigma} - g^{\alpha\rho}\,J^{\beta\sigma} - g^{\beta\sigma}\,J^{\alpha\rho} + g^{\alpha\sigma}\,J^{\beta\rho})
\end{aligned}
$$

- Two Casimir operators: (a) $P_\mu P^\mu$

  (b) $W_\mu W^\mu = -m^2 s(s+1)$

- Pauli-Lubanski vector: $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}J_{\nu\rho}P_\lambda$

Coleman Mandula Theorem

- Theorem: Only possible CONSERVED quantities that transform as TENSORS under the Lorentz group are the generators of the Poincaré algebra $(P^\mu, J^{\mu\nu})$ and Lorentz scalars S_a (internal symmetry).

- Tensors: $a_{\mu_1\mu_2\ldots\mu_N}$, combinations of Lorentz vector indices

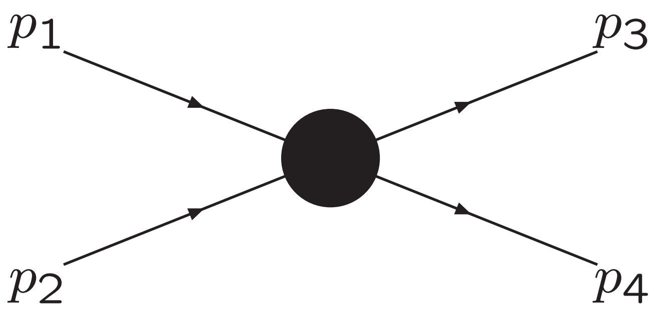
  $\Longrightarrow$ Tensors are bosons

- Loop-hole in theorem: conserved charges transforming as spinors

  $\Longrightarrow$ Supersymmetry

Heuristic Argument

- Consider $2 \longrightarrow 2$ spinless scattering



- For simplicity $p_i^2 = m_i^2 = m^2$

- Momentum conservation (Lorentz symmetry): $p_1 + p_2 = p_3 + p_4$

- The scattering amplitude $\mathcal{M}$ is a Lorentz scalar

  $\Longrightarrow$ can only depend on Lorentz invariants of p_i

- Mandelstam variables: $s,\ t \longrightarrow\ E_{\text{cm}} = \sqrt{s},\ \theta$, scattering angle

- Now postulate an additional **external** symmetry
- For fixed $E_{\text{cm}}\,(\sqrt{s})$ this must put furthr restrictions on θ
- However, only discrete angles θ are allowed
- But $\mathcal{M}$ is analytic (observation)

$$\Longrightarrow \mathcal{M}(\theta) = \text{const.}$$

- <u>Example:</u> Consider (traceless) Tensor $R_{\mu\nu} = p_\mu p_\nu - \frac{1}{4}m^2$
- $R_{\mu\nu}$ is conserved $\Longrightarrow$ only $\theta = 0, \pi$ allowed

Supersymmetry Algebra

- Two-component fermionic generators: $Q_\alpha, \, Q^\dagger_{\dot\alpha}$

- Additional algebra relations:

$$\{Q_\alpha, \, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, \, Q^\dagger_{\dot\beta}\} = 0$$

$$\{Q_\alpha, \, Q^\dagger_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$[P^\mu, \, Q_\alpha] = [P^\mu, \, Q^\dagger_{\dot\alpha}] = 0$$

$$[J^{\mu\nu}, \, Q_\alpha] = -(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$

- Note $\sigma^{\mu\nu} \equiv \frac{i}{4}(\sigma^\mu \overline{\sigma}^\nu - \sigma^\nu \overline{\sigma}^\mu)$

- Graded Lie Algebra

Supersymmetry: Simple Consequences

- $[Q_\alpha, T^j] = 0$

  $\Longrightarrow$ gauge quantum numbers unchanged

- $[Q_\alpha,\ P^\mu] = 0 \Longrightarrow [Q_\alpha, P_\mu P^\mu] = [Q_\alpha, (\text{Mass})^2] = 0$

  $\Longrightarrow$ Mass unchanged; $\Longrightarrow$ Supersymmetry must be broken

- $[Q_\alpha, W^\mu] = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu (\sigma_{\rho\sigma})_\alpha{}^\beta Q_\beta$

  $\Longrightarrow$ $Q_\alpha,\ Q^\dagger_{\dot\alpha}$ change spin

- Can show: change spin by $\pm\frac{1}{2}$

- Thus supersymmetry is an **external** symmetry

- Graded Lie algebra, Coleman-Mandula is evaded

Supersymmetry Motivation

Aesthetics:

- Haag-Łopuszanski-Sohnius:

> Supersymmetry is the unique external symmetry extension of the Poincaré algebra with non-trivial S-matrix

Scale:

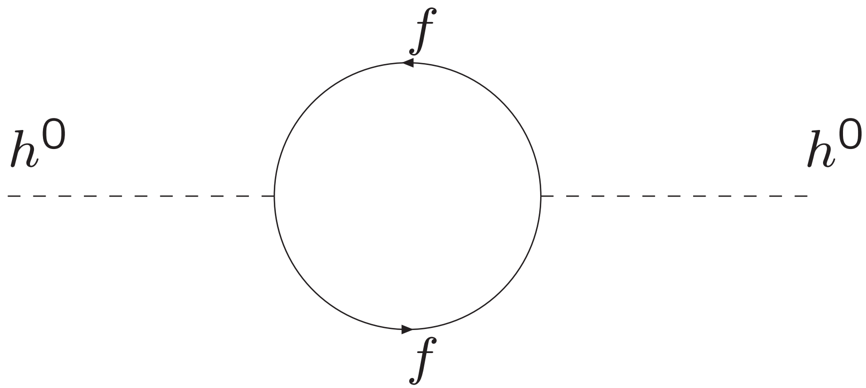
- Electron: $e_L, \, e_R \longrightarrow$ scalar electron: $\tilde{e}_L, \, \tilde{e}_R$, spin=0

- Experiment: Mass($\tilde{e}_{L,R}$) $\gg$ Mass(e)

- What can we say about the possible scale of Mass($\tilde{e}_{L,R}$)?

  $\longrightarrow$ Hierarchy problem

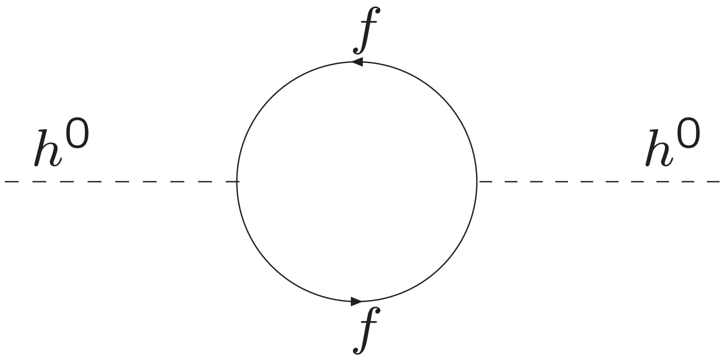
Hierarchy Problem: the Higgs Mass

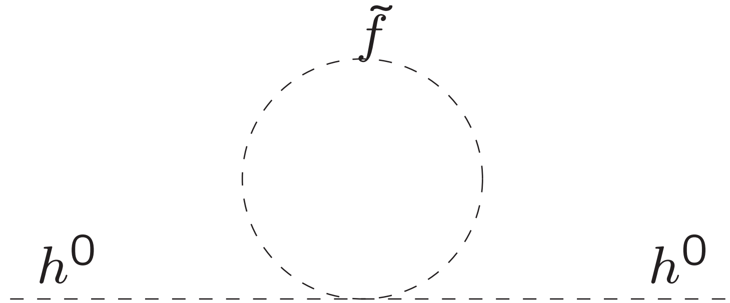


$$-i\Pi^{f}_{hh} = -\frac{N(f)\lambda_f^2}{8\pi^2}\left[\Lambda^2 + 3m_f^2 \ln\left(\frac{\Lambda^2 + m_f^2}{m_f^2}\right) + 2m_f^2 \frac{\Lambda^2}{\Lambda^2 + m_f^2}\right]$$

- Leading term is quadratically divergent $\Longrightarrow$ very sensitive to high scale
- In QED, corrections to electron mass only have logarithmic divergences
- Who cares? If I renormalize, these divergences never appear
- But if there is a new scale of physics (M_{GUT}, M_{Pl}?) which couples to the Higgs mass: this also feeds in quadratically via the finite correction
- There is no hierarchy problem if the SM is valid for all energies

Supersymmetric Solution





$$\Pi_{hh}^{f+\tilde{f}}(0) = i\frac{\lambda_f^2 N(f)}{16\pi^2} \left[-2m_f^2 \left(1 - \log\frac{m_f^2}{\mu^2}\right) + 4m_f^2 \log\frac{m_f^2}{\mu^2} \right.$$
$$\left. +2m_{\tilde{f}}^2 \left(1 - \log\frac{m_{\tilde{f}}^2}{\mu^2}\right) - 4m_f^2 \log\frac{m_{\tilde{f}}^2}{\mu^2} - |A_f|^2 \log\frac{m_{\tilde{f}}^2}{\mu^2} \right].$$

- Quadratic divergence cancels
- If $m_{\tilde{f}}$ too large, again have "quadratic divergence"
- Expect $m_{\tilde{f}} = \mathcal{O}(1\,\text{TeV})$ if supersymmetry exists $\Longrightarrow$ LHC!!

SUPERSYMMETRIC STANDARD MODEL

External Symmetries	**Internal Symmetries**
Poincaré Symmetry (Space-time)	Gauge Symmetries SU(3) x SU(2) x U(1)
Supersymmetry	

- In Standard Model also have accidental global symmetries:

  Baryon Number & Lepton Number

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- Particle content $\longrightarrow$ double + extra Higgs doublet

SUSY SPECTRUM

Standard Model + SUSY $\Longrightarrow$ Double Spectrum (+2 Higgs Doublets)

e^- (spin $= \frac{1}{2}$)	$\longleftrightarrow$	$\tilde{e}$ ($s = 0$)	scalar electron
top t ($s = \frac{1}{2}$)	$\longleftrightarrow$	$\tilde{t}$ ($s = 0$)	scalar top
$H^\pm$ ($s = 0$)	$\longleftrightarrow$	$\tilde{H}^\pm$ ($s = \frac{1}{2}$)	Higgsino
H^0, h^0 ($s = 0$)	$\longleftrightarrow$	$\tilde{H}^0$, $\tilde{h}^0$ ($s = \frac{1}{2}$)	Higgsino
$W^\pm$ ($s = 1$)	$\longleftrightarrow$	$\tilde{W}^\pm$ ($s = \frac{1}{2}$)	Wino
B, W^0 ($s = 1$)	$\longleftrightarrow$	$\tilde{B}$, $\tilde{W}^0$ ($s = \frac{1}{2}$)	Bino, neutral Wino
$g_{a=1,\ldots,8}$ ($s = 1$)	$\longleftrightarrow$	$\tilde{g}_a$ ($s = \frac{1}{2}$)	Gluino

Higgs Sector

- Fermionic Higgs $\longrightarrow$ Chiral Anomalies

(• holomorphic superpotential & Fermion masses)

- Two Higgs doublets: H_1, H_2 (H_u, H_d)

- Eight real Higgs fields; 3 absorbed in Higgs mechanism

- 5 observable Higgs fields

CP-even: h^0, H^0,, CP-odd: A^0, Charged: $H^\pm$

- Parameters: $\mu H_1 H_2$, $\tan\beta = \frac{v_2}{v_1}$

- Quartic coupling is fixed by supersymmetry

SUPERSYMMETRY BREAKING

- Supersymmetry: Mass(e^-) = Mass($\tilde{e}^-_{L,R}$) $\Longrightarrow$ Susy must be broken

- No agreed model of supersymmetry breaking $\longrightarrow$ phenomenological ansatz

- Must preserve solution to hierarchy problem

Breaking term		**Examples**
$M_{1/2}\,\chi\chi$	$\longrightarrow$	$M_1 \tilde{B}\tilde{B}$, $\;M_2\, \tilde{W}^{0,\pm}\tilde{W}^{0,\mp}$, $\;M_3\, \tilde{g}\tilde{g}$
$M_0^2\, \phi^\dagger\phi$	$\longrightarrow$	$m^2_{\tilde{e}L}\tilde{e}^\dagger_L\tilde{e}_L$, $m^2_{\tilde{e}R}\tilde{e}^\dagger_R\tilde{e}_R$, $m^2_{\tilde{u}L}\tilde{u}^\dagger_L\tilde{u}_L$, $m^2_{\tilde{u}R}\tilde{u}^\dagger_R\tilde{u}_R$,
$A_{ijk}\, \phi_i\phi_j\phi_k$	$\longrightarrow$	$A^e_{ij}\begin{pmatrix}\tilde{\nu}_i\\ \tilde{e}_i\end{pmatrix}_L h_1\tilde{e}_{jR}$, $\quad A^d_{ij}\begin{pmatrix}\tilde{u}_i\\ \tilde{d}_i\end{pmatrix}_L h_1\tilde{d}_{jR}$
$B_{ij}\, \phi_i\phi_j$	$\longrightarrow$	$B\, h_1 h_2$

Parameters of the Supersymmetric SM

- SM: 19 parameters (massless neutrinos)
- Now: two extra parameters in Higgs sector $(\tan\beta, \mu)$

SUSY BREAKING

- A^e_{ij}, A^d_{ij}, A^u_{ij} $\longrightarrow$ 27 real + 27 phases
- $\mathbf{M}^2_{\tilde{Q}}$, $\mathbf{M}^2_{\tilde{U}}$, $\mathbf{M}^2_{\tilde{D}}$, $\mathbf{M}^2_{\tilde{L}}$, $\mathbf{M}^2_{\tilde{E}}$ $\longrightarrow$ 30 real + 15 phases
- M_1, M_2, M_3 $\longrightarrow$ 3 real, 1 phase

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124 parameters!!

Supersymmetry Breaking

- Constrained MSSM or minimal Supergravity: breaking is universal at GUT scale

- Universal scalar masses: $\mathbf{M}^2_{\tilde{Q}}, \mathbf{M}^2_{\tilde{U}}, \mathbf{M}^2_{\tilde{D}}, \mathbf{M}^2_{\tilde{L}}, \mathbf{M}^2_{\tilde{E}} \longrightarrow M_0^2$ @ M_{GUT}

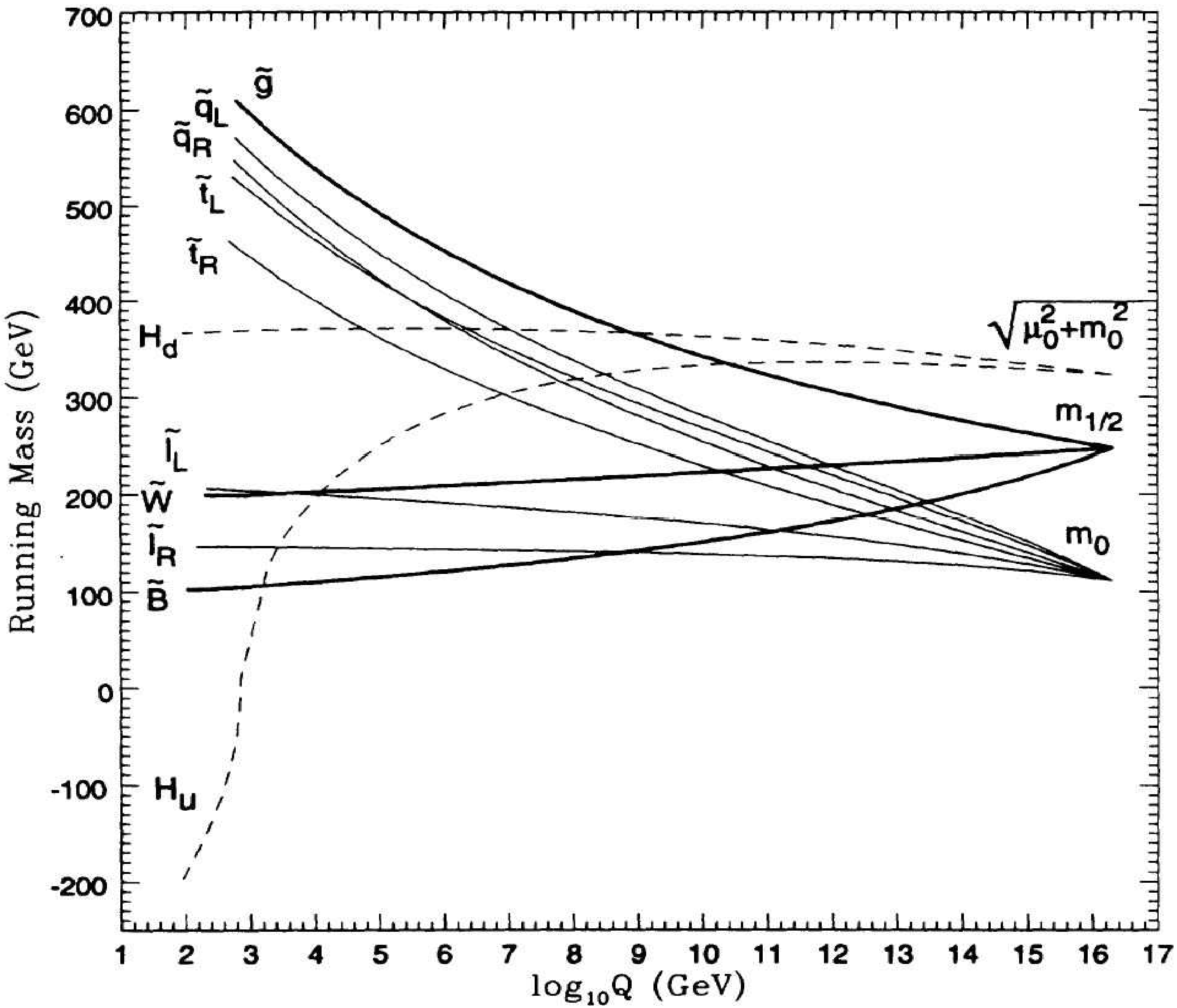
- Universal gaugino masses: $M_1, M_2, M_3 \longrightarrow M_{1/2}$ @ M_{GUT}

- Universal trilinears: $A^e_{ij}, A^d_{ij}, A^u_{ij} \longrightarrow A \cdot h^e_{ij}, A \cdot h^d_{ij}, A \cdot h^u_{ij}$ @ M_{GUT}

- Parameters

$$\boxed{M_0,\ M_{1/2},\ A,\ B,\ \mu,\ \tan\beta}$$

Supersymmetric Spectrum (Kane et al.)



Radiative Electroweak Symmetry Breaking

- Obtain the correct scale of electroweak symmetry breaking

- This fixes the parameter B as well as $|\mu|$

- Left with 4 1/2 parameters

$$\boxed{M_0,\ M_{1/2},\ A,\ \mathrm{sgn}(\mu),\ \tan\beta}$$

$\mathbf{SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}}$ Breaking

- Below Mass scale M_W: $SU(3)_C \times U(1)_{EM}$ are good symmetries
- Only colour and electric charge are good gauge quantum numbers
- Matter and Radiation can mix!
- In particular: $\tilde{H}^{\pm}(s=\frac{1}{2})$ and $\tilde{W}^{\pm}(s=\frac{1}{2})$ can mix
- And they do!

Chargino Mixing

- Mass Matrix in $(\tilde{W}^{\pm}, \tilde{H}^{\pm})$-basis:

$$\mathcal{M}_{\pm} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \sin\beta & \mu \end{pmatrix}$$

- $\tan\beta = \frac{v_2}{v_1}$: Ratio of H_1 and H_2 vacuum expectation values

- μ: Higgs mixing parameter, $\mu H_1 H_2$;

- Mass Eigenstates: $\tilde{\chi}^{\pm}_{i=1,2}$

- Chargino pair-production is the main SUSY search modes at e^+e^-–colliders

- Charged leptons do not mix: lepton-number conservation

Neutralino Mixing

- Similarly: Bino ($\tilde{B}$), neutral Wino ($\tilde{W}^3$), and neutral Higgsinos ($\tilde{H}^0_{1,2}$) can mix.

- Mass Mixing Matrix in ($\tilde{B}$, $\tilde{W}^3$, $\tilde{H}_1$, $\tilde{H}_2$) basis

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & -M_z \cos\beta \sin\theta_w & M_z \sin\beta \sin\theta_w \\ 0 & M_2 & M_z \cos\beta \cos\theta_w & -M_z \sin\beta \cos\theta_w \\ -M_z \cos\beta \sin\theta_w & M_z \cos\beta \cos\theta_w & 0 & \mu \\ M_z \sin\beta \sin\theta_w & -M_z \sin\beta \cos\theta_w & \mu & 0 \end{pmatrix}$$

- $\sin\theta_w$: electroweak mixing

- Mass Eigenstates: $\tilde{\chi}^0_{i=1,2,3,4}$

- $\tilde{\chi}^0_1$: lightest neutralino

Scalar Fermion Mixing

- The left- and right scalar fermions also mix, eg A-term

$$A h_t \tilde{t}_L \tilde{t}_R h_2^0 \quad \longrightarrow \quad A h_t v_2 \tilde{t}_L \tilde{t}_R$$

- For example the scalar top (stop) mass matrix squared

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_t^2 + m_{\tilde{t}L}^2 + (\frac{1}{2} - \frac{2}{3} s_W^2) \cos 2\beta M_Z^2 & -m_t(A + \mu \cot\beta) \\ -m_t(A + \mu \cot\beta) & m_t^2 + m_{\tilde{t}R}^2 + \frac{2}{3} s_W^2 \cos 2\beta M_Z^2 \end{pmatrix}$$

- Mixing is only large for third generation

Superfields

- Can combine complex scalar field and two-component fermion field into single entity with definite supersymmetry transformation properties: chiral superfield

$$\Phi(x,\theta) = \phi(x) + \sqrt{2}\theta^\alpha \psi_\alpha(x) + \theta^\alpha \theta_\alpha F(x)$$

- F is an auxiliary field

- Examples:

(a) singlet electron

$$E^c(x,\theta) = \tilde{e}^\star_R(x) + \sqrt{2}\theta\psi_{eR}(x) + \theta^\alpha\theta_\alpha F^*_{\tilde{e}R}(x)$$

(b) lepton doublet

$$L(x,\theta) = \begin{pmatrix} \tilde{\nu}_L(x) + \sqrt{2}\theta\psi_{\nu L}(x) + \theta\theta F_{\tilde{\nu}L}(x) \\ \tilde{e}_L(x) + \sqrt{2}\theta\psi_{eL}(x) + \theta\theta F_{\tilde{e}L}(x) \end{pmatrix}$$

Superfields – Superpotential

- Similarly for the other fields

$$\boxed{L_i,\ E_i^c,\ Q_i,\ U_i^c,\ D_i^c,\ H_1,\ H_2}$$

- Product of chiral superfields is again a chiral superfield

- Combine into holomorphic function called the superpotential

$$W = \sum \left[a_i \Phi_i + b_{ij} \Phi_i \Phi_j + c_{ijk} \Phi_i \Phi_j \Phi_k \right]$$

- $\Phi^\dagger$ not allowed

- Can extract all interactions between the chiral superfields from Superpotential: pure scalar interactions and Yukawa interactions

- What is the superpotential of the supersymmetric SM?

Supersymmetric SM Superpotential

- Must consider all gauge invariant terms up to power three

$$\boxed{W_1 = (h_e)_{ij}\, L_i H_1 E_j^c + (h_d)_{ij}\, Q_i H_1 D_j^c + (h_u)_{ij}\, Q_i H_2 U_j^c + \mu H_1 H_2}$$

- These give mass to quarks and leptons, generalization of SM terms

- There are more gauge invariant terms

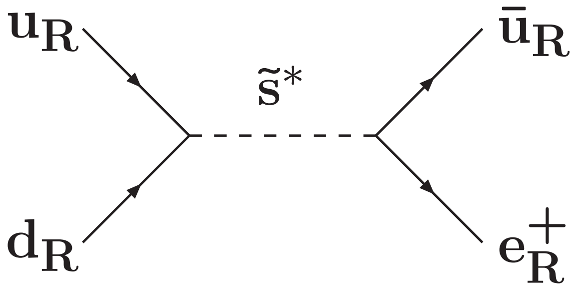
$$\boxed{W_2 = \underbrace{\lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \kappa_i L_i H_2}_{} + \underbrace{\lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k}_{}}$$

Lepton Number Violating Baryon Num. Viol.

- Together these lead to rapid proton decay

PROTON DECAY

- **Together LQD and UDD lead to Rapid Proton Decay:** $p \rightarrow \pi^0 e^+$



- **Resulting in the Strict Bound:**

$$\boxed{\lambda'_{i1j} \cdot \lambda''_{11j} < 2 \cdot 10^{-27} \left(\frac{M_{\tilde{d}_j}}{100\ \text{GeV}} \right)^2 , \quad i = 1, 2\,,\ j \neq 1\,,}$$

- **Therefore the SUSY-SM as Defined is Experimentally Excluded**

- At least one Coupling must be Zero: Guaranteed by a Symmetry

- Discrete symmetry; global symmetry; gauge symmetry, GUT

SYMMETRIES

- Focus here on simplest case: Discrete Symmetry
- Conventional choice to solve proton decay problem: **R-PARITY**

$$R_p = (-1)^{3B+L+2S}, \qquad\qquad R_p\, e^- = (-1)^{3B+L+2S}\, e^- = (+1)\, e^-$$

- Physically equivalent: **MATTER PARITY, M_2**

$$\begin{aligned} (L, E^c, Q, U^c, D^c) &\longrightarrow -(L, E^c, Q, U^c, D^c) \\ (H_1, H_2) &\longrightarrow +(H_1, H_2) \end{aligned}$$

- Prohibit all dim$\leq$ 4 lepton & baryon-number viol. operators:

LLE, LQD, UDD, LH

- Convention: **MSSM= SUSY-SM constrained by matter parity (M_2)**
- But allows dangerous dim-5 proton decay operators: $\frac{1}{\Lambda}QQQL$

OTHER SIMPLE OPTIONS

- **BARYON PARITY:** Prohibits UDD Terms

$$(Q, U^c, D^c) \longrightarrow -(Q, U^c, D^c)$$
$$(L, E^c, H_1, H_2) \longrightarrow +(L, E^c, H_1, H_2)$$

- **LEPTON PARITY:** Prohibits LLE, LQD, LH Terms

$$(L, E^c) \longrightarrow -(L, E^c)$$
$$(Q, U^c, D^c, H_1, H_2) \longrightarrow +(Q, U^c, D^c, H_1, H_2)$$

- **BARYON TRIALITY (B_3):** Prohibits UDD Terms

$$\Psi_j \rightarrow e^{i\alpha_j 2\pi/3}\Psi_j$$

	Q	U^c	D^c	L	E^c	H_d	H_u
α_j	0	2	1	2	2	2	1

- Or higher discrete symmetry $\mathbf{Z}_N$

Analysis of Discrete Symmetries

- `Krauss & Wilczek:` expect all global symmetries to be violated by quantum gravity effects, also for discrete symmetries

- Exception: discrete symmetry is the remnant of a spontaneously broken gauge symmetry

  $\Longrightarrow$ "discrete gauge symmetry"

- `Ibanez & Ross:` if the original U(1) gauge symmetry is anomaly-free

  $\Longrightarrow$ conditions on the remnant discrete symmetry

  $\Longrightarrow$ "anomaly-free discrete symmetry"

- `Ibanez & Ross:` systematic study of all $\mathbf{Z}_{2,3}$ with MSSM particle content

  $\Longrightarrow$ only two anomaly-free discrete symmetries: R_p, B_3

- R_p: dangerous dim-5 proton decay operators

General Analysis

Christoph Luhn, Marc Thormeier, HD

- We extended the Ibanez & Ross analysis to all $\mathbf{Z}_N$ symmetries

- Find four $\mathbf{Z}_6$, nine $\mathbf{Z}_9$, and nine $\mathbf{Z}_{18}$ as new fundamental anomaly-free symmetries

- Require:

  1. $\mu H_1 H_2$ in Lagrangian

  2. No dim-5 proton decay operators

  3. See-saw neutrino mass term: LH_2LH_2

- Only **proton-hexality**: P_6, and **baryon-triality**: B_3 remain

- P_6 same as R_p, except prohibits dangerous dim-5 terms

SUPERPOTENTIAL

- The P_6 superpotential is given by

$$\boxed{W_{P_6} = (h_e)_{ij}\, L_i H_1 E_j^c + (h_d)_{ij}\, Q_i H_1 D_j^c + (h_u)_{ij}\, Q_i H_2 U_j^c + \mu H_1 H_2}$$

- P_6 is **THE** symmetry of the MSSM

- The B_3 superpotential is given by

$$\boxed{W_{\mathsf{B}_3} = W_{\mathsf{P}_6} \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \kappa_i L_i H_2}$$

- Which is preferable? Does it matter?

P_6 versus B_3

- What are the main differences?

- Lightest Supersymmetric Particle (LSP):

  - P_6: LSP is stable

CMSSM: • Typically: LSP $= \tilde{\chi}_1^0$, the lightest neutralino

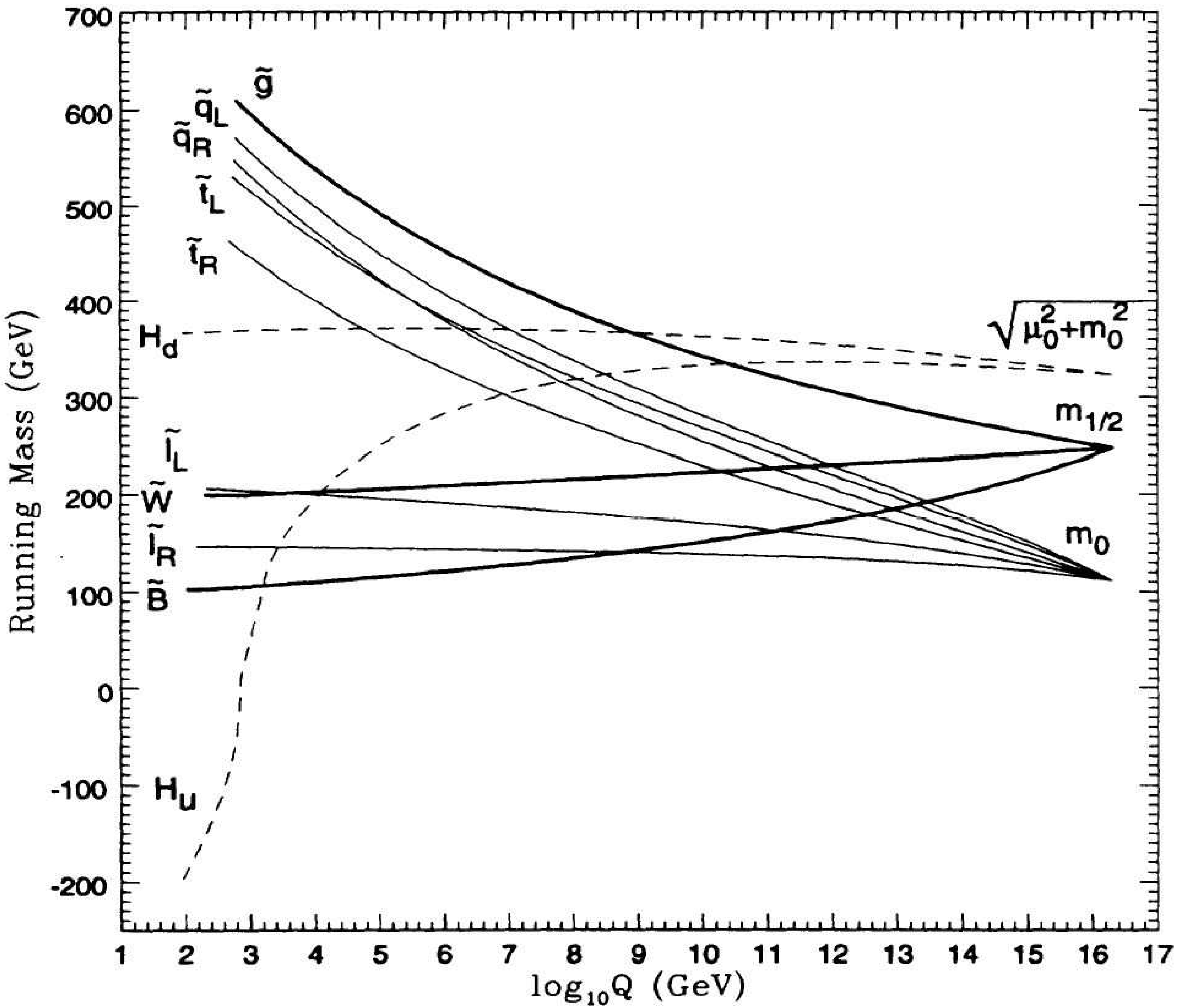
- $R_p\, \tilde{\chi}_1^0 = (-1)^{3B+L+2S} \tilde{\chi}_1^0 = (-1) \tilde{\chi}_1^0$

- $\tilde{\chi}_1^0$ is R-parity odd $\Longrightarrow$ must be stable

- $\tilde{\chi}_1^0$ gives missing E_T signatures in colliders

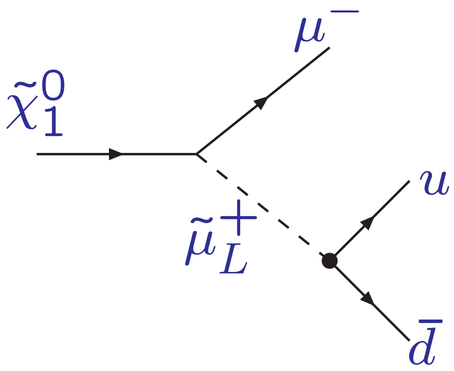
- LSP is a WIMP $\Longrightarrow$ LSP is ideal dark matter candidate

Supersymmetric Spectrum (Kane et al.)



P_6 versus B_3: LSP

- B_3: LSP is unstable $\Longrightarrow$ can't be dark matter



- Need alternativ dark matter candidate $\Longrightarrow$ e.g. axino or gravitino

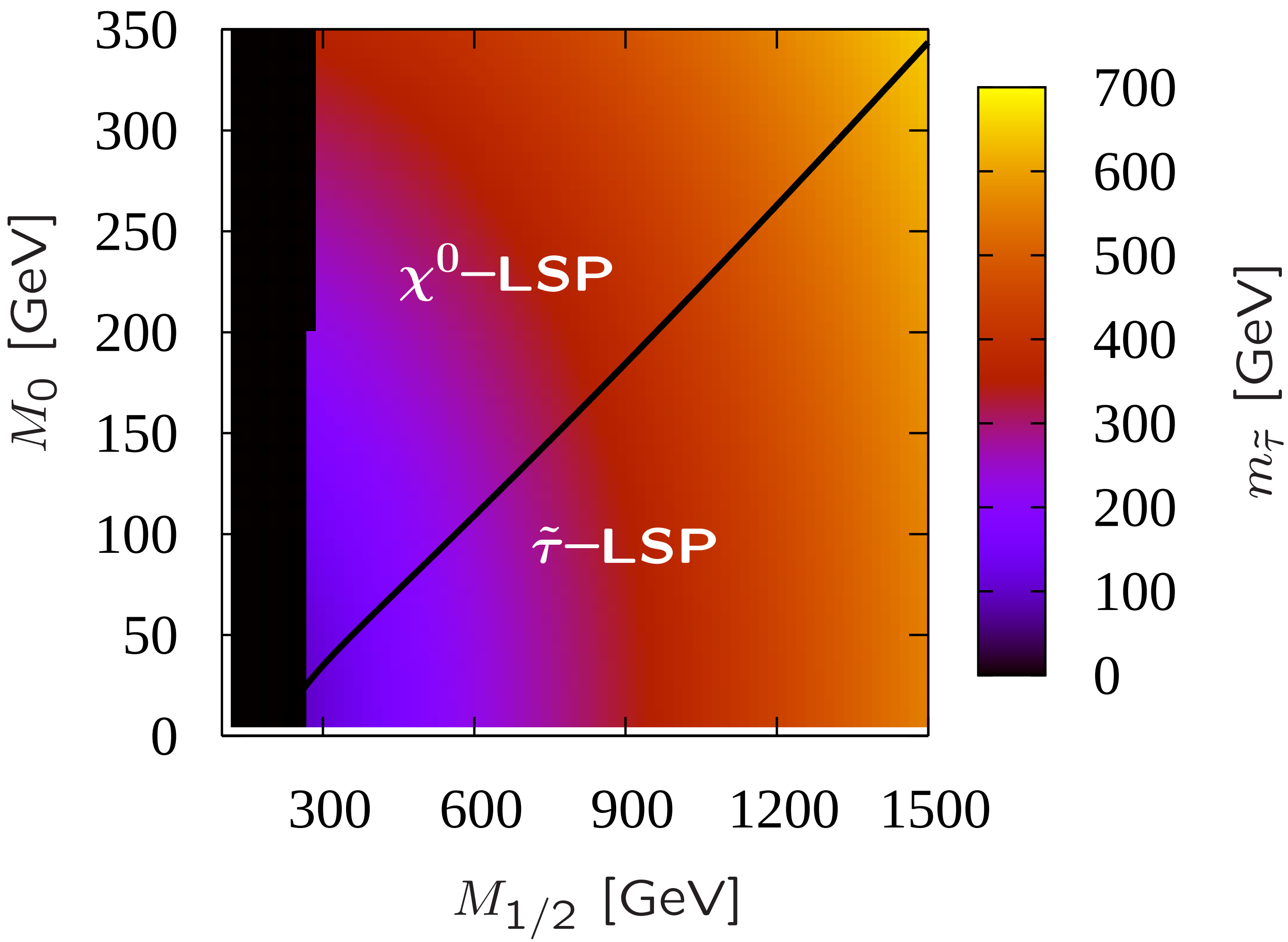
  J.E. Kim et al; L. Covi et al

- (Many) Extra leptons in the final state from LSP decay

- No cosmological constraint: LSP need not be $\tilde{\chi}_1^0$

$$\text{LSP} \in \{\chi_1^0,\, \chi_1^+,\, \tilde{\nu}_L,\, \tilde{\ell}^{\pm}_{L,R},\, \tilde{\tau}_1^{\pm},\, \tilde{t}_1,\, \tilde{q}_{L,R},\, \tilde{g}\}$$

- Actually in CMSSM (even with zero RPV) get stau LSP, for example



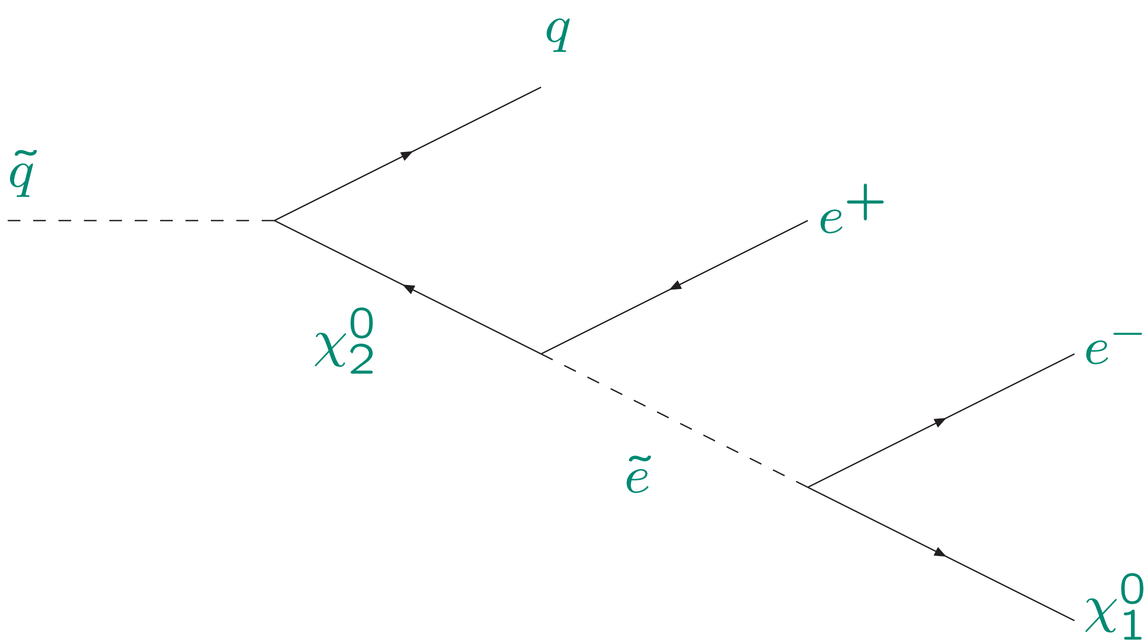
HD, S. Grab

- Can also get $\tilde{\nu}$ in wide regions of parameter space

- And sometimes $\tilde{e}$, $\tilde{\mu}$

SUSY Cascade

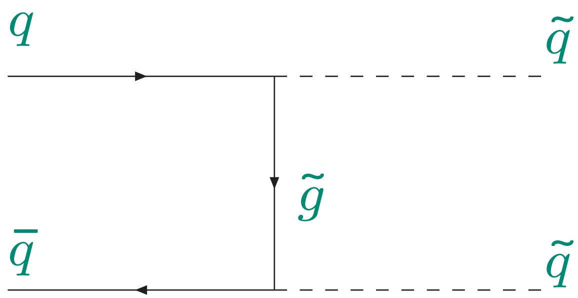
- Independent of cosmology: nature of LSP is essential for collider searches
- Cascade Decay



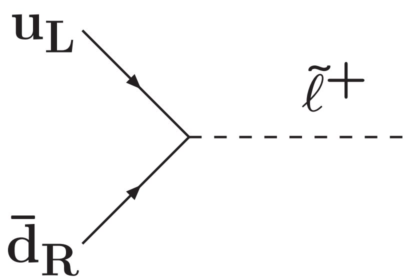
- These decays happen "instantaneously" in detector
- If SUSY exists: LSP appears in every event
- P_6: missing E_T; B_3: extra leptons

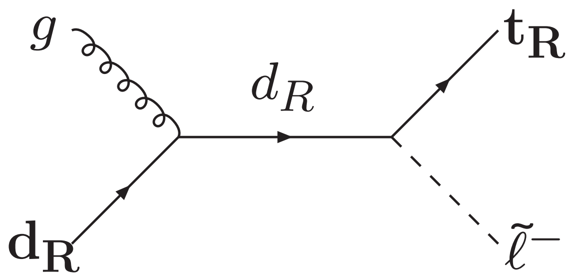
P_6 versus B_3: SUSY Production

- P_6: only pair production is possible: need R-parity even final state



- B_3: Resonant/Associated Single SUSY Production possible



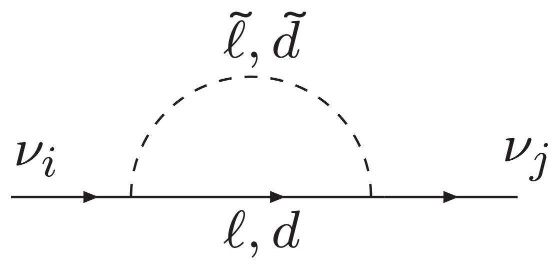


- Larger kinematic reach
- X-section proportional to λ'^2

Neutrino Masses

- P_6
  - Must introduce right-handed neutrino chiral superfield
  - Need extra physics scale for see-saw mechanism: $10^{10} - 10^{12}$ GeV

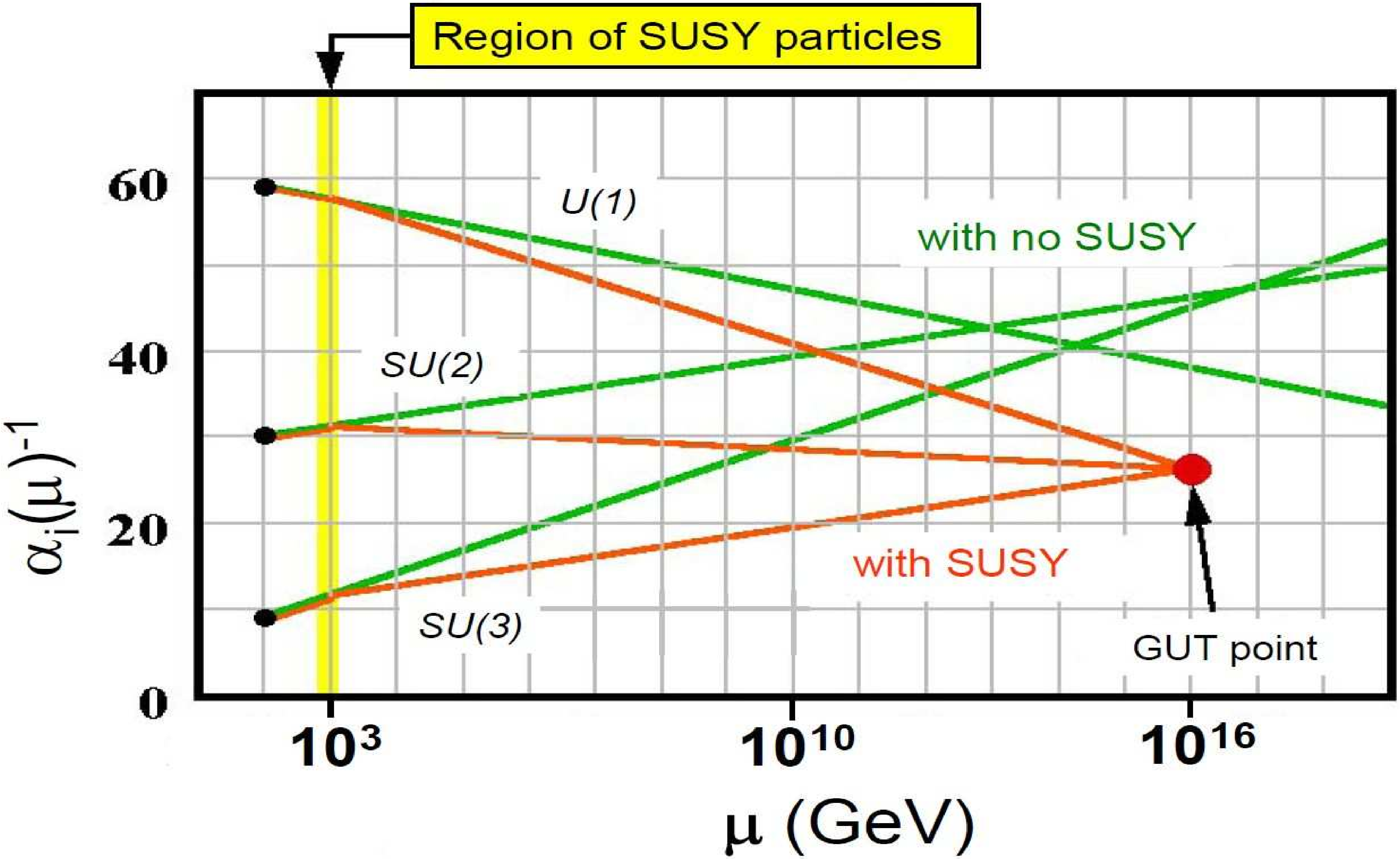
- B_3
  - Through $\kappa_i L_i H_2$ neutrinos and neutralinos mix $\rightarrow$ one massive neutrino
  - Other neutrino mass via loop correction



  - Need a mechanism for small $\kappa_i = \mathcal{O}(10\,\text{MeV})$

    $\longrightarrow$ B_3 mSUGRA w/ universal SUSY breaking

Unification



Unification

- Gauge couplings meet in Supersymmetry at $M_{\mathrm{GUT}} = \mathcal{O}(10^{16}\,\mathrm{GeV})$
- Independent of P_6 vs B_3
- Just recall: B_3 treats quarks and leptons differently

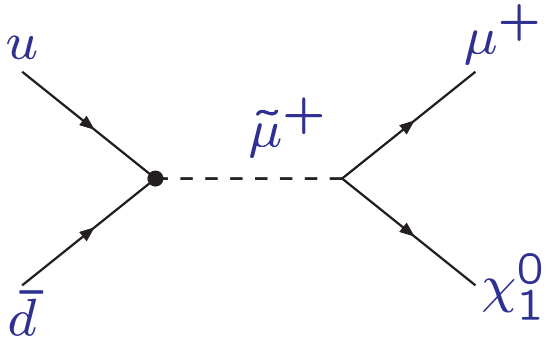
  $\Longrightarrow$ Not compatible with GUT symmetry where quarks & leptons in same multiplet

- But can generate B_3-terms after GUT breaking (eg Brahm and Hall)
- B_3 is compatible with string unification (eg Bento, Hall and Ross)

One Example SUSY Search: B_3

Resonant Slepton Production – χ_1^0 LSP

Richardson, Seymour, HD
Grab, Krämer, Trenkel, HD



- The neutralino is Majorana: $\chi_1^0 \to \mu^\pm + 2\,\text{jets}$

$\Longrightarrow$ Potential Signature: 2 like-sign muons

- Background: What are potential SM sources of $\mu^+\mu^+$ or $\mu^-\mu^-$?

SM $\mu^+\mu^+$ Background

- WZ, ZZ-Production: $W^\pm \to \mu^\pm \nu_\mu$, $Z^0 \to \mu^+\mu^+$

- $t\bar{t}$-Production: $t \to W^+ b$ (one lepton from b-decay)

- $b\bar{b}$-production (1 b–quark hadron oscillates: $B^0 \leftrightarrow \bar{B}^0$)

$$b \to c + (W^-)^* \to c + \mu^- + \bar{\nu}_\mu$$

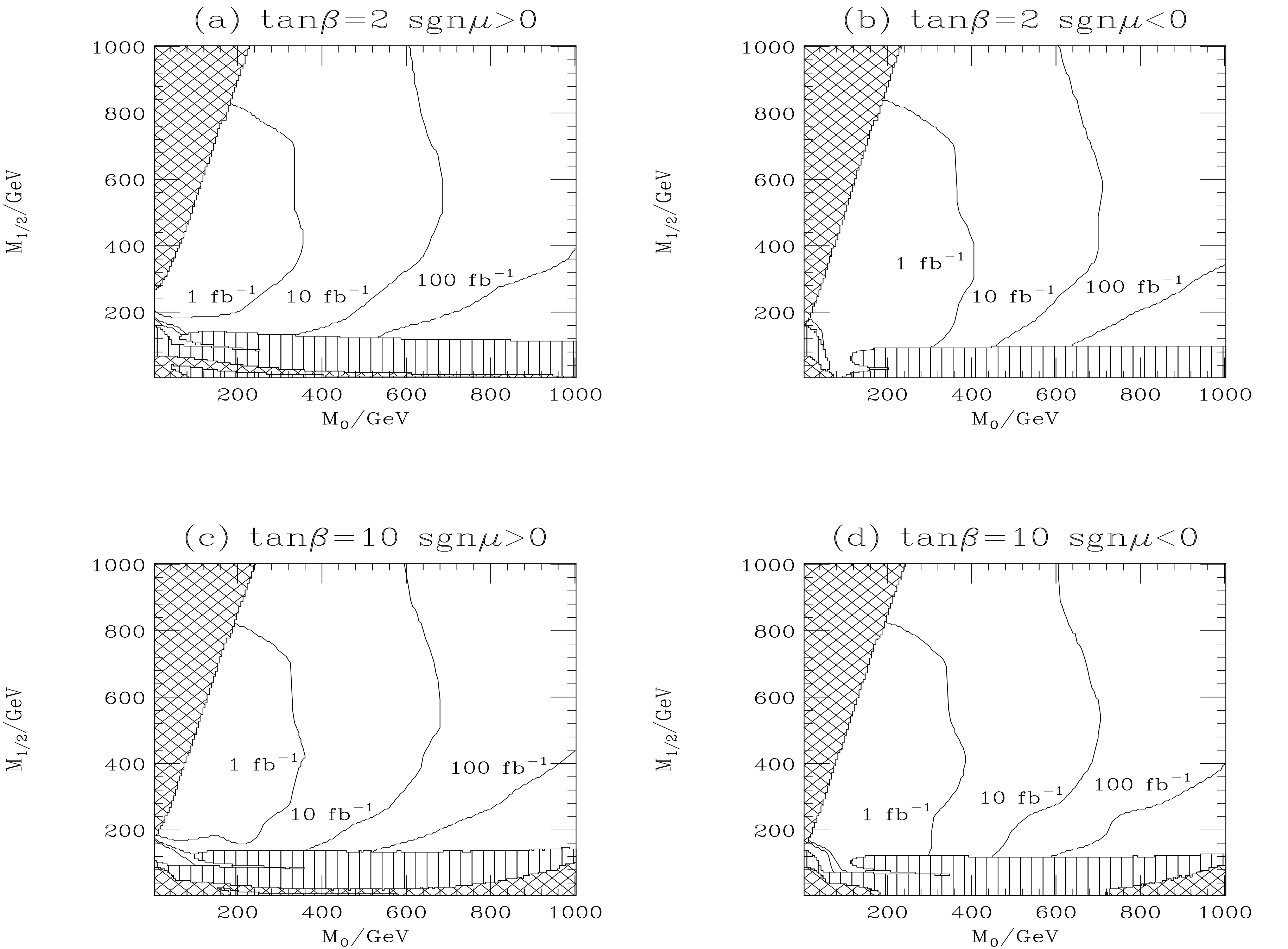
- Single top production (*i.e.* $t\bar{b}$-production)

- Non-physics background (experimental effects, fake id's etc)

CUTS: Signal vs Background

- leptons: $|\eta| < 2.0$
- like-sign leptons: $p_T^{\text{lepton}} \geq 40\,\text{GeV}$
- Isolated leptons: $E_T < 5\,\text{Gev}$ in $R = \sqrt{\Delta\phi^2 + \Delta\eta^2} = 0.4$
- Reject events with: $60\,\text{GeV} < M_T < 85\,\text{GeV}$, applied to both leptons
- Veto events with like-sign same flavour leptons if $p_T > 10\,\text{GeV}$ and passes isolation cuts
- Only allow small missing $E_T < 20\,\text{GeV}$
- Note: this is only a theory analysis. Proper analysis at LHC includes a full simulation of detector

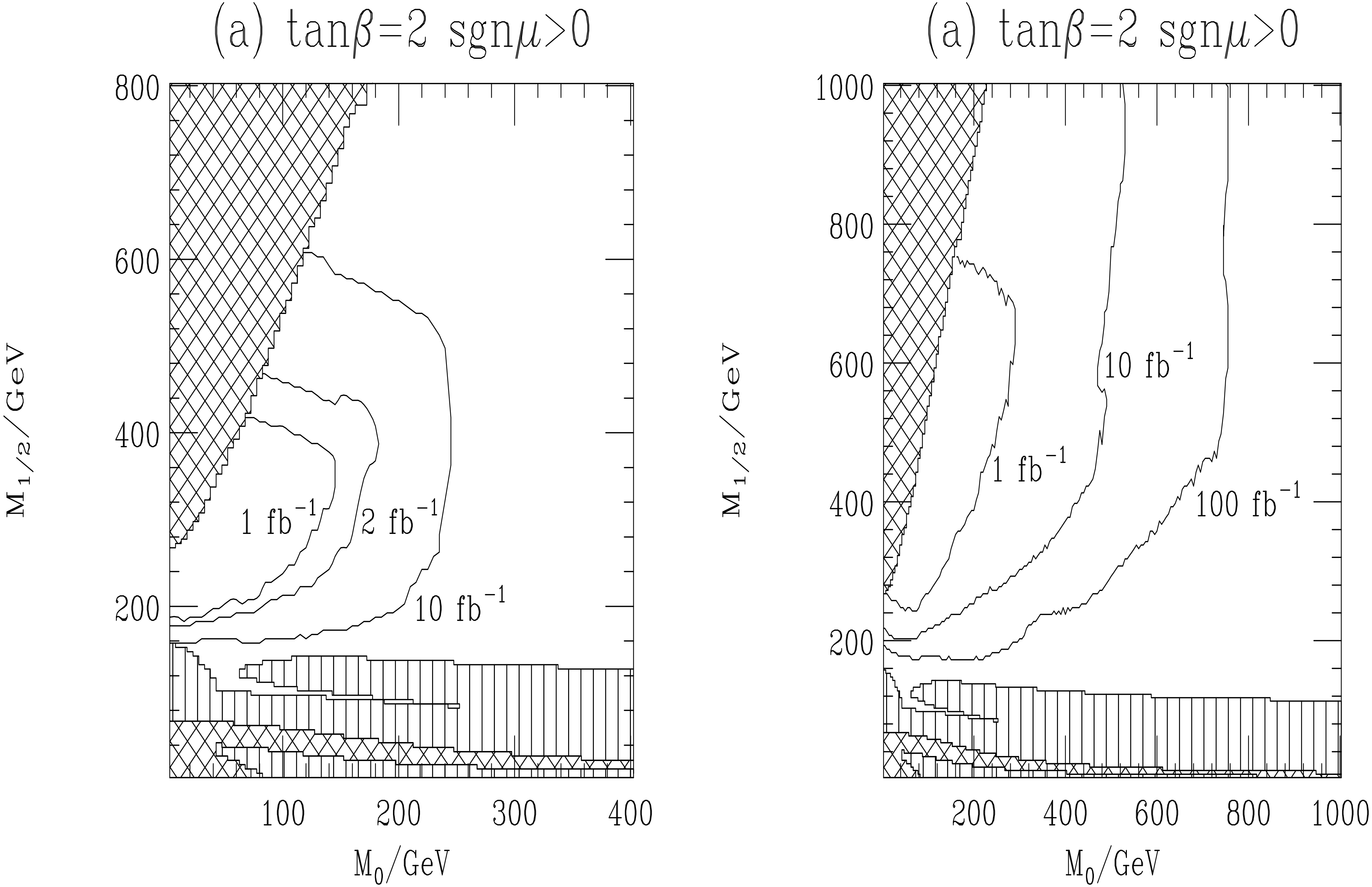
Background Process	Number of Events			
	After p_T cut	After isolation and p_T cuts	After isolation, p_T, M_T, $\not{E}_T$ cuts and OSSF lepton veto.	After all cuts
WW	3.6 ± 0.5	0.0 ± 0.06	0.0 ± 0.06	0.0 ± 0.06
WZ	239 ± 2.5	198.6 ± 2.3	3.8 ± 0.3	3.8 ± 0.3
ZZ	55.4 ± 0.7	45.2 ± 0.6	1.04 ± 0.09	1.04 ± 0.09
$t\bar{t}$	$(4.4 \pm 0.2) \times 10^3$	0.28 ± 0.13	0.06 ± 0.06	0.06 ± 0.06
$b\bar{b}$	$(4.4 \pm 0.9) \times 10^4$	0.0 ± 1.6	0.0 ± 1.6	0.0 ± 1.6
Single Top	36.6 ± 1.5	0.0 ± 0.004	0.0 ± 0.004	0.0 ± 0.004
Total	$(4.9 \pm 0.9) \times 10^4$	244.1 ± 2.9	4.9 ± 1.6	4.9 ± 1.6


(a) tanβ=2 sgnμ>0
(b) tanβ=2 sgnμ<0
(c) tanβ=10 sgnμ>0
(d) tanβ=10 sgnμ<0
1 fb⁻¹
10 fb⁻¹
100 fb⁻¹
M1/2/GeV
M0/GeV
200
400
600
800
1000

SUSY is also Background

- if SUSY already discovered: other SUSY production mechanisms are also background

- Extra SUSY Cut: veto all events when there are more than two jets, with each $p_T > 50\,\text{GeV}$

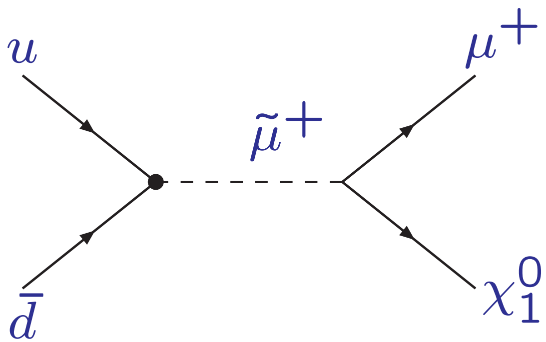


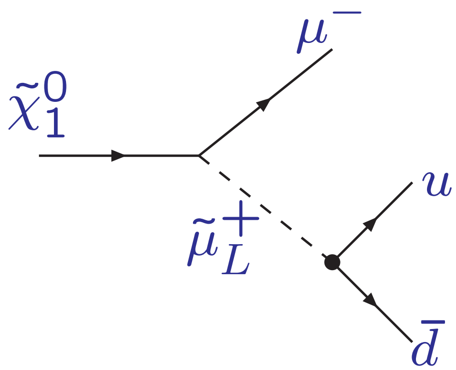
Tevatron Reach

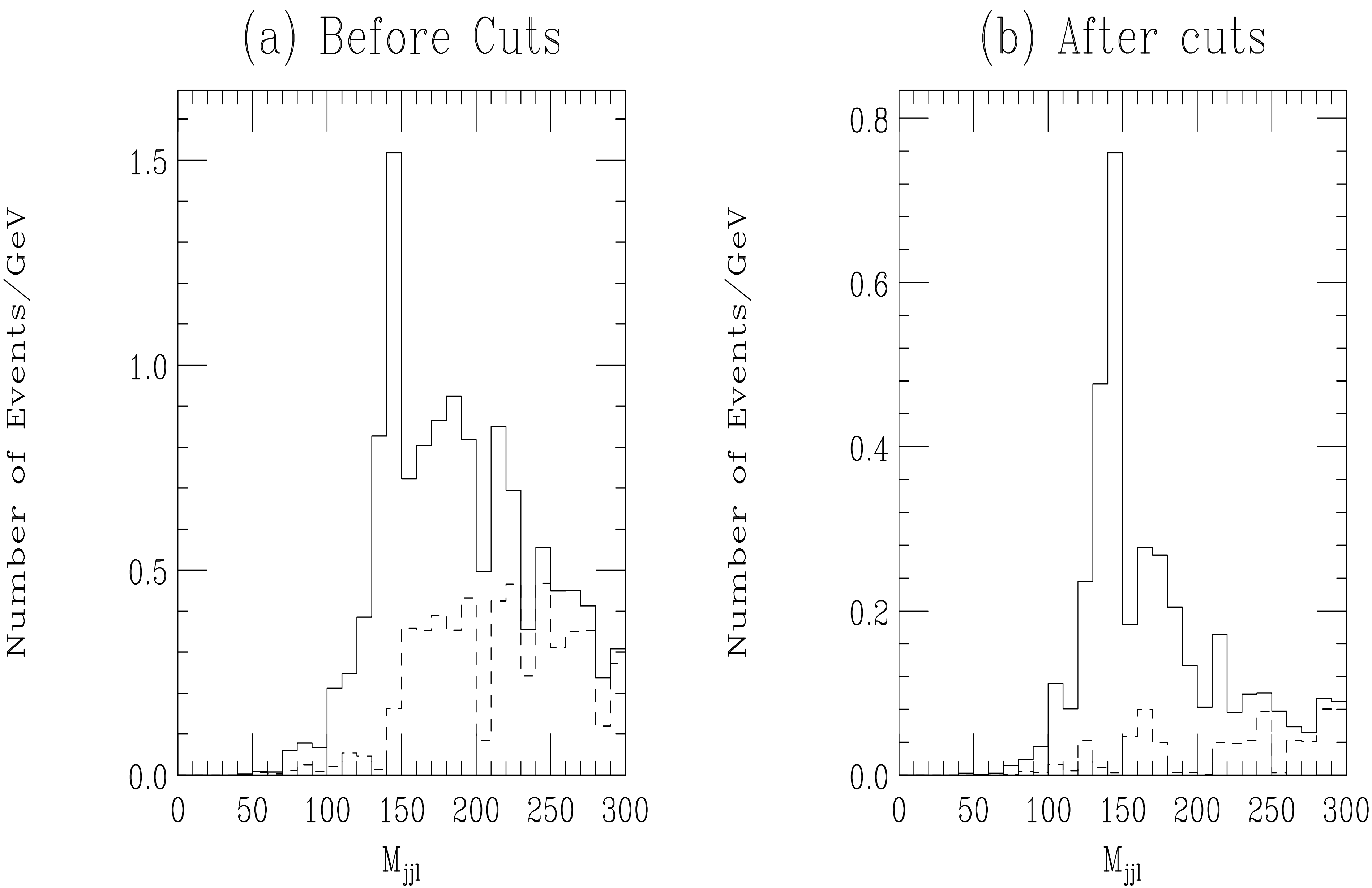
LHC Reach

Including SUSY Backgd, and non-resonant like-sign di-lepton sources, $\lambda' = 0.01$

- If discovered, can also try to measure mass of particles, eg neutralino



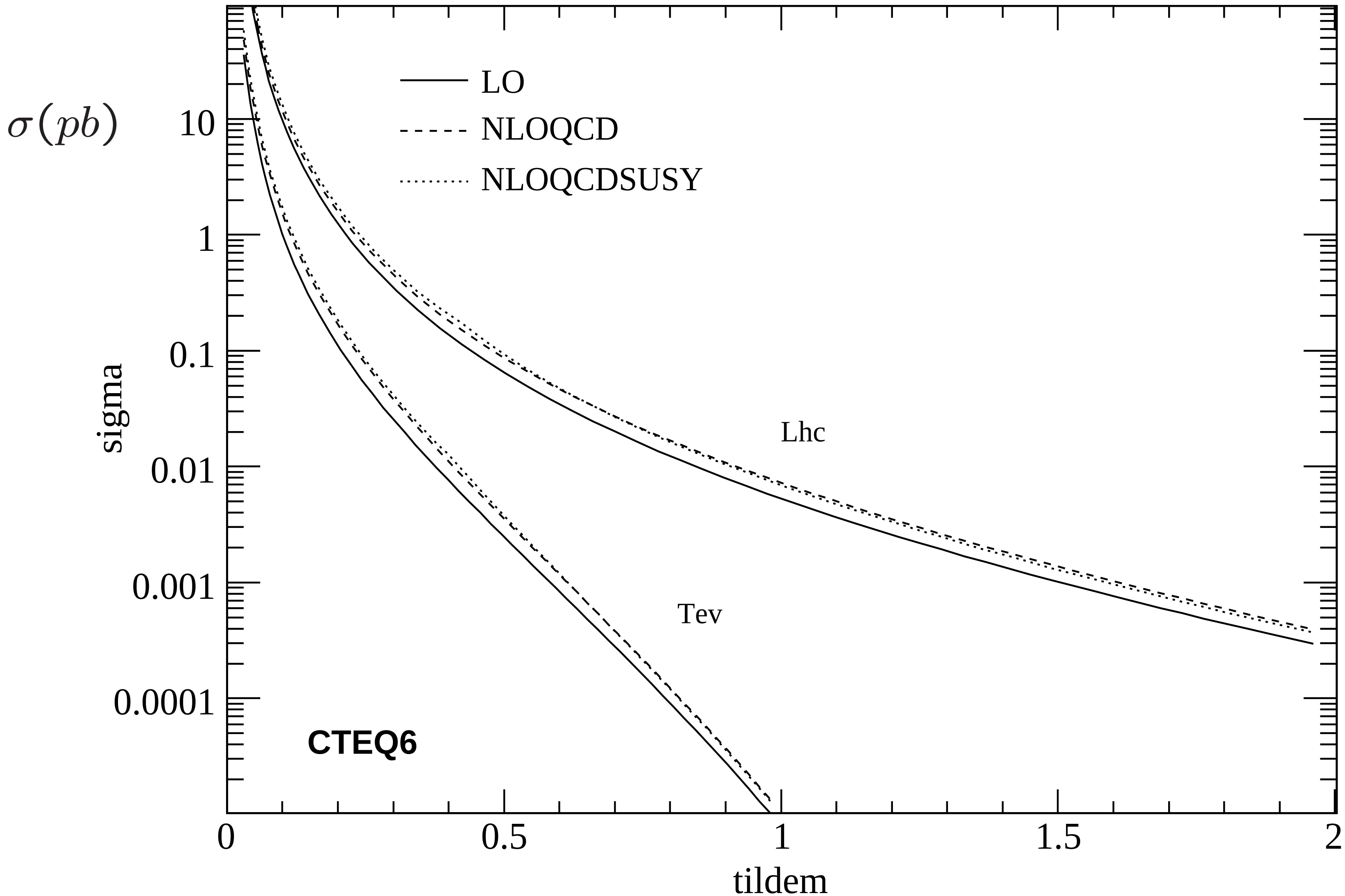


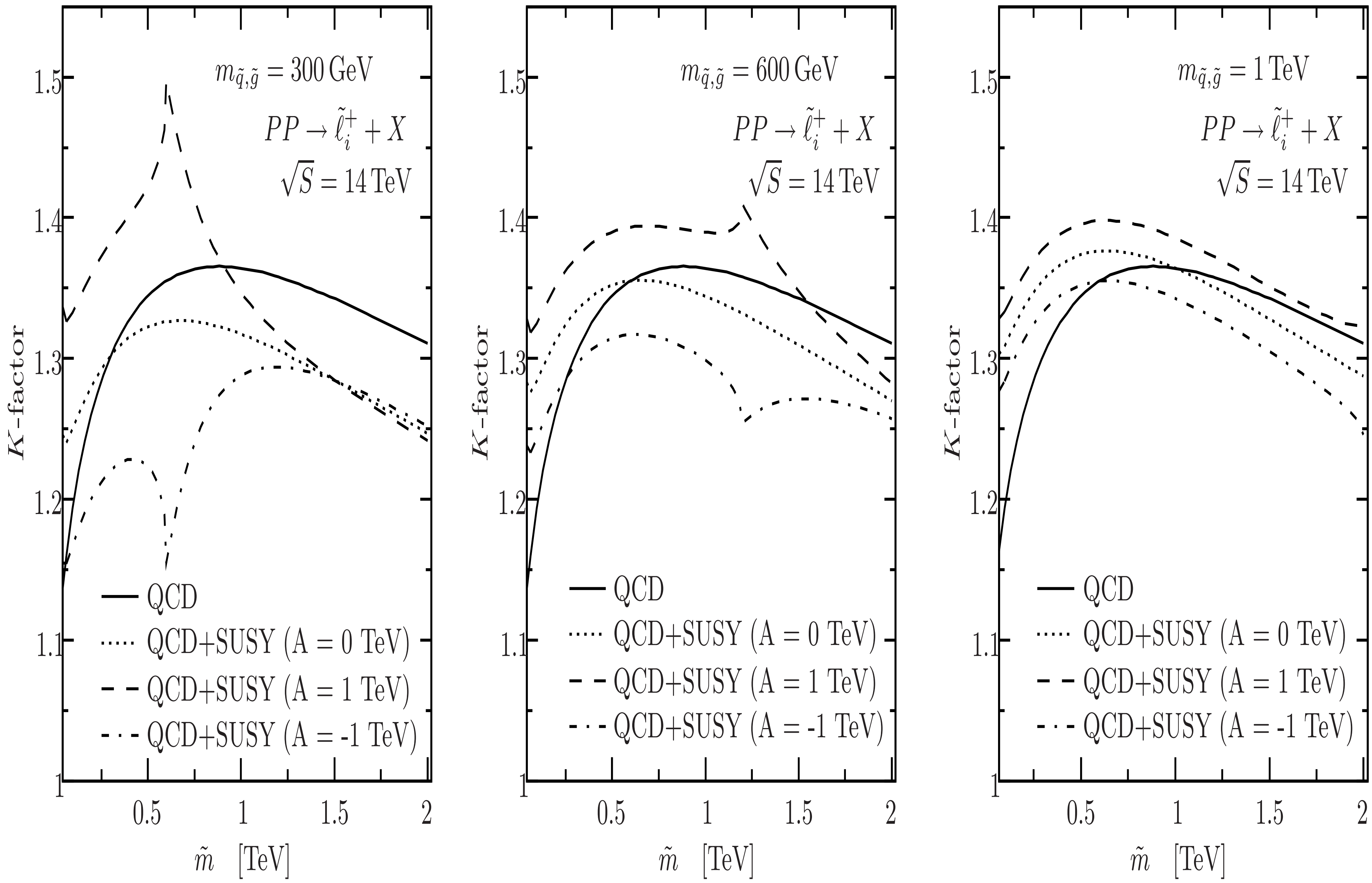


Neutralino Mass Determination at the LHC, $\mathcal{L} = 10\ \text{fb}^{-1}$

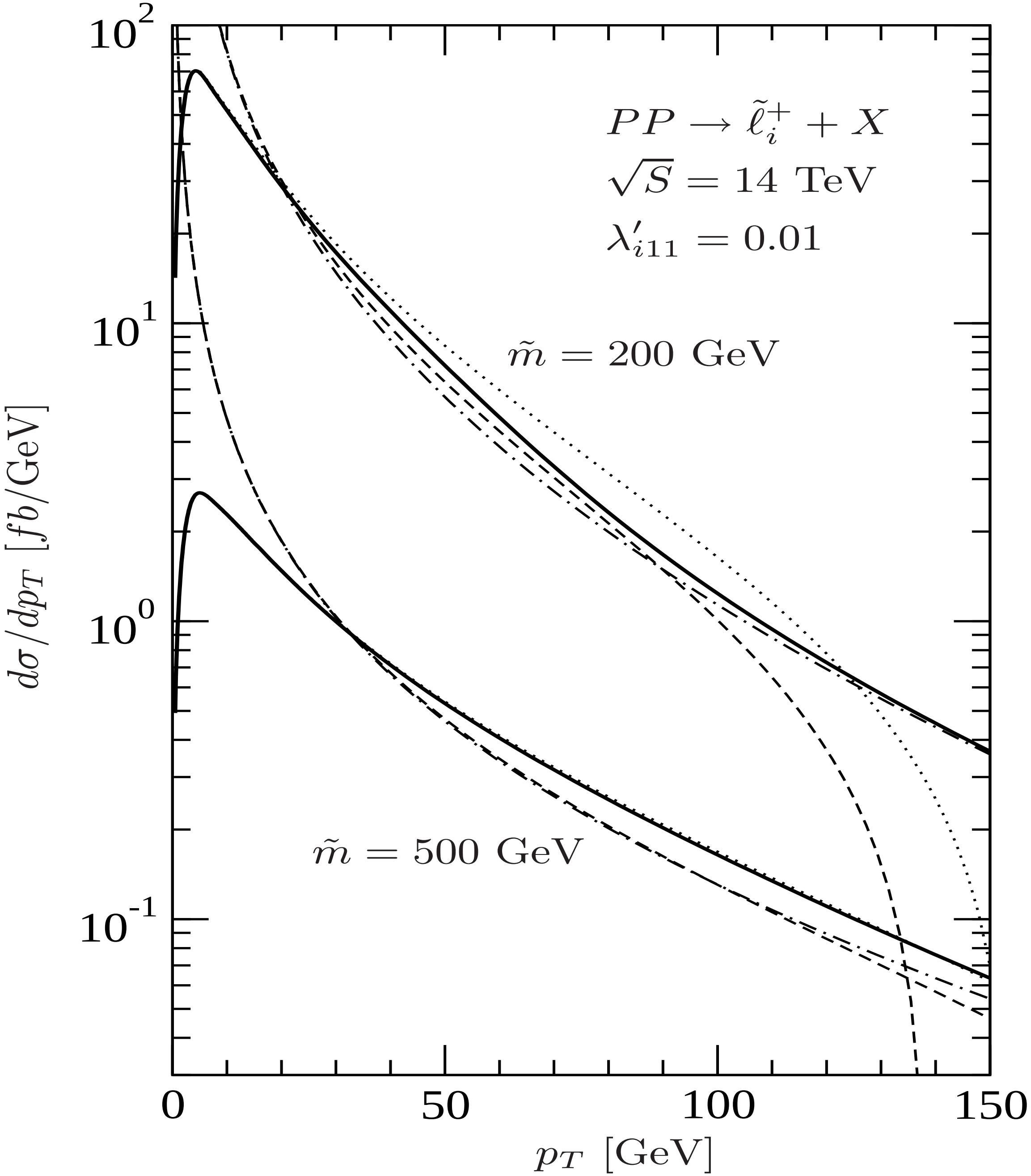
- Computed radiative corrections to resonant slepton production:

Grab, Krämer, Trenkel, HD





$m_{\tilde{q},\tilde{g}} = 300\,\text{GeV}$
$PP \to \tilde{\ell}_i^+ + X$
$\sqrt{S} = 14\,\text{TeV}$
$m_{\tilde{q},\tilde{g}} = 600\,\text{GeV}$
$m_{\tilde{q},\tilde{g}} = 1\,\text{TeV}$
K-factor
$\tilde{m}$ [TeV]
QCD
QCD+SUSY (A = 0 TeV)
QCD+SUSY (A = 1 TeV)
QCD+SUSY (A = -1 TeV)
1
1.1
1.2
1.3
1.4
1.5
0.5
1
1.5
2



$PP \rightarrow \tilde{\ell}_i^+ + X$
$\sqrt{S} = 14$ TeV
$\lambda'_{i11} = 0.01$
$\tilde{m} = 200$ GeV
$\tilde{m} = 500$ GeV
$d\sigma/dp_T$ [fb/GeV]
p_T [GeV]
10^2
10^1
10^0
10^{-1}
0
50
100
150

β
$\bar{\nu}_\tau$
$\tilde{\chi}^0$
u
d
s
τ^-
ν_μ
M_Z
ν_e
α
g
e^-