Parton Shower Monte Carlos

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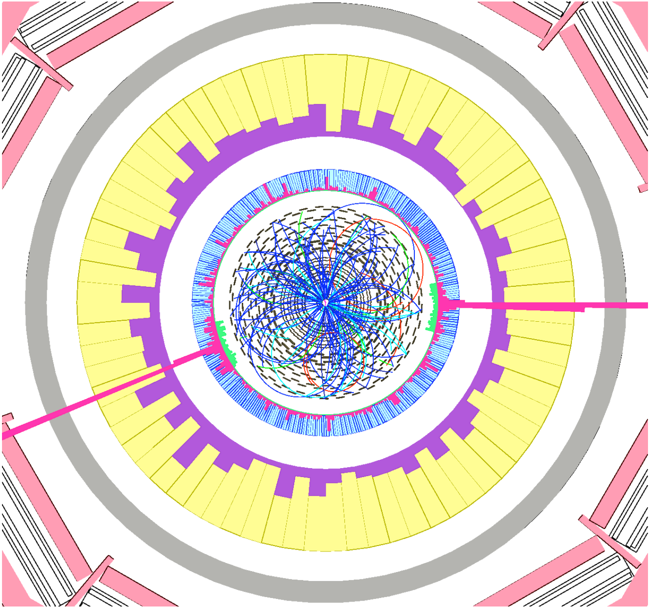
DESY Theory Workshop, Sept 29 – Oct 2, 2009

Outline

- Introduction.
- Next Generation MC programs.
- NLO matching.

Can you spot the Higgs?



Why Monte Carlos?

We want to understand

$$\mathscr{L}_{\text{int}} \longleftrightarrow \text{Final states} \; .$$

Why Monte Carlos?

LHC experiments require
sound understanding of signals and *backgrounds*.

↑

Full detector simulation.

↑

Fully exclusive hadronic final state.

↑

Monte Carlo event generator with
parton shower, hadronization model, decays of unstable particles.

↑

Parton level computations.

Monte Carlo Event Generators

- Complex final states in full detail (jets).
- Arbitrary observables and cuts from final states.
- Studies of new physics models.

- Rates and topologies of final states.
- Background studies.
- Detector Design.
- Detector Performance Studies (Acceptance).

- *Obvious* for calculation of observables on the quantum level
$$|A|^2 \longrightarrow \text{Probability}.$$

pp Event Generator

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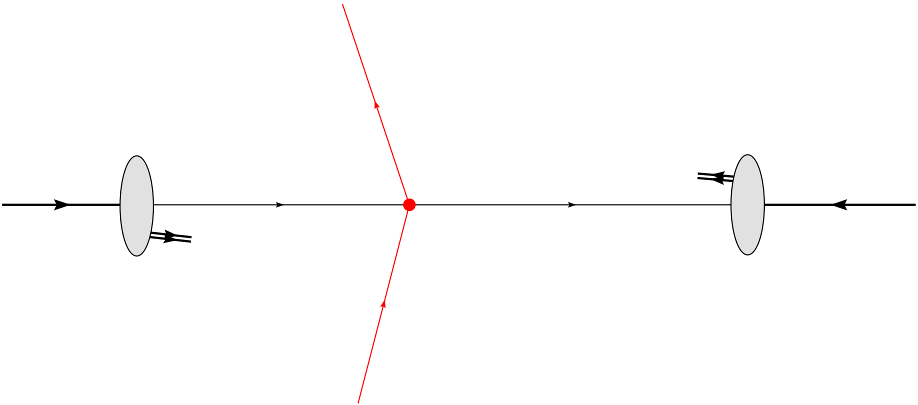
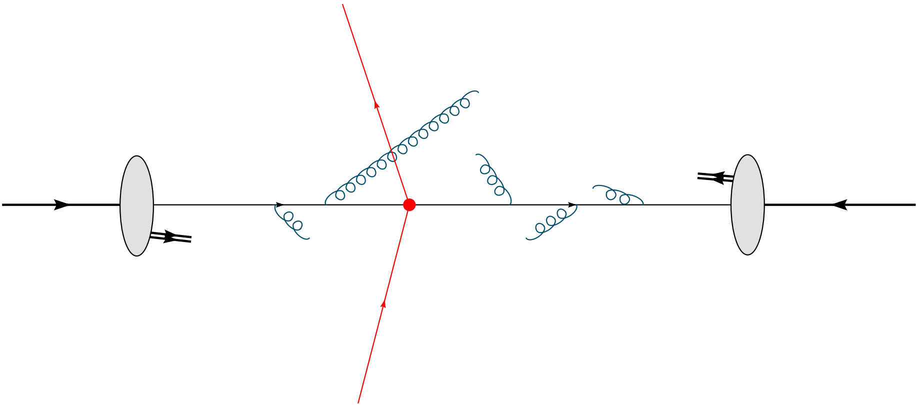


pp *Event Generator*



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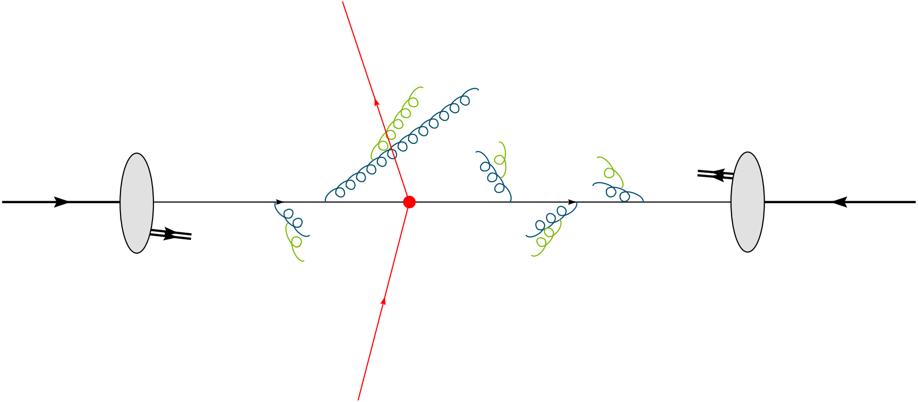

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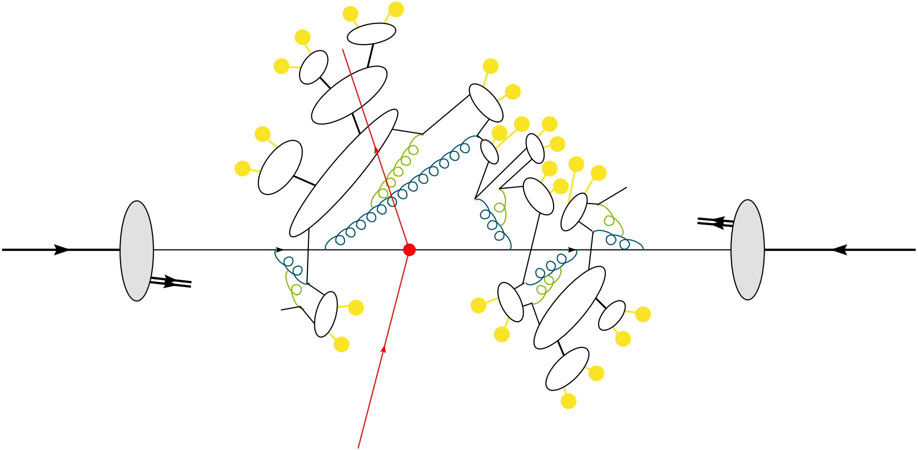

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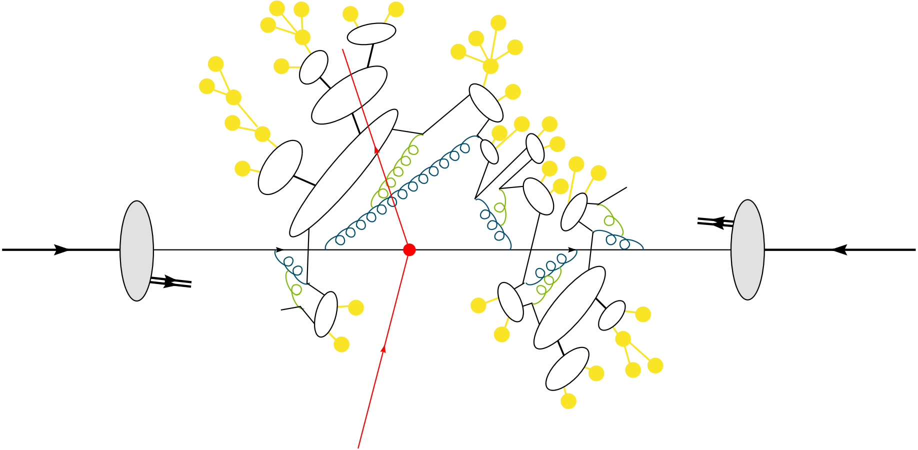

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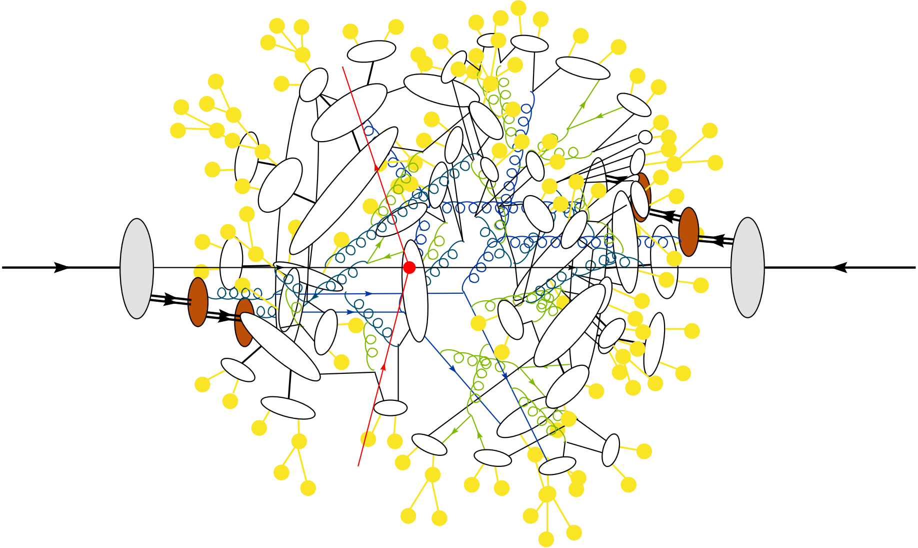

Divide and conquer

Partonic cross section from Feynman diagrams

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{hard}}\mathrm{d}P(\mathrm{partons} \to \mathrm{hadrons})$$

Note, that

$$\int \mathrm{d}P(\mathrm{partons} \to \mathrm{hadrons}) = 1 \ ,$$

- σ remains unchanged
- introduce realistic fluctuations into distributions.

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- introduce realistic fluctuations into distributions.

Simulation steps governed by different scales
$\longrightarrow$ separation into ($Q_0 \approx 1\,\text{GeV} > \Lambda_{\rm QCD}$)

$$\begin{aligned} dP(\text{partons} \to \text{hadrons}) = \; & dP(\text{resonance decays}) && [\Gamma > Q_0] \\ & \times dP(\text{parton shower}) && [\text{TeV} \to Q_0] \\ & \times dP(\text{hadronisation}) && [\sim Q_0] \\ & \times dP(\text{hadronic decays}) && [O(\text{MeV})] \end{aligned}$$

Divide and conquer

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Quite complicated integration.

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Quite complicated integration.

Monte Carlo is the only choice.

Next Generation MC programs

- New programs for the LHC era have been developed
  - SHERPA
  - Pythia8
  - Herwig++
- Pythia8 and Herwig++ now recommended as the better options
  (where the physics has been implemented already).
- Already widely used in LHC collaborations
  (still along with FORTRAN versions).
- Slowly becoming the default choice
  always when new developments are involved.

Full manual. Detailed documentation and many examples on wiki pages.

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DOI 10.1140/epjc/s10052-008-0798-9

THE EUROPEAN PHYSICAL JOURNAL C

Special Article - Tools for Experiment and Theory

Herwig++ physics and manual

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Abstract In this paper we describe Herwig++ version 2.2, a general-purpose Monte Carlo event generator for the simulation of hard lepton-lepton and hadron-hadron collisions. A number of important hard scattering processes are available, together with an interface via the Les Houches Accord to specialized matrix element generators for additional processes. The simulation of Beyond the Standard Model (BSM) physics includes a range of models and allows new models to be added by encoding the Feynman rules of the model. The parton-shower approach is used to simulate initial- and final-state QCD radiation, including colour

Some Herwig++ features

- Many SM hard processes available. BSM cascades with spin correlations. LH files.
- BSM physics included.
- New parton shower working in IS, FS, t–decays.

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Matching NLO computations and parton showers

The problem:
Consider n and $n+1$ body ME

$$|M_n^{(0)}|^2 \qquad 2\mathrm{Re} M_n^{(0)} M_n^{(1)} \qquad |M_{n+1}^{(0)}|^2 \; .$$

- Both present in NLO as Born+Virtual and Real ME.
- Parton shower adds $n+1$ st emission as well (accurate to leading log accuracy).

$\Rightarrow$ Potential double counting!

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Two popular approaches:

- MC@NLO
- POWHEG

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
x = real emission phase space, Born, Observable, Real, Virtual.

$$\langle O\rangle_{\mathrm{NLO}} = BO(0) + VO(0) + \int_0^1 dx\, \frac{O(x)R(x)}{x},$$

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x = real emission phase space, Born, Observable, Real, Virtual.

$$\langle O \rangle_{\text{NLO}} = BO(0) + VO(0) + \int_0^1 dx \, \frac{O(x)R(x)}{x} \,,$$

Add/subtract soft/collinear piece $A(x)$ ($\lim_{x\to 0} A(x) = R(x)$):

$$\langle O \rangle_{\text{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx \, \frac{O(x)R(x) - O(0)A(x)}{x} \,,$$

where

$$\bar{V} = V + \int_0^1 dx \, \frac{A(x)}{x} = \text{IR finite} \,.$$

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \qquad .$$

From Born $\otimes$ zero/one parton shower emission:

$$\langle O \rangle_{\mathrm{PS}} = \int dx\, O(x) \left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu) \right]$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \approx 1 - \int dx \, \frac{P(x)}{x} \, .$$

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$$\begin{aligned} \langle O \rangle_{\mathrm{PS}} &= \int dx \, O(x) \left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu) \right] \\ &= BO(0) \left[1 - \int_\mu \frac{dx}{x} P(x) \right] + \int_\mu dx \, O(x) B \frac{P(x)}{x} \, . \end{aligned}$$

PS@NLO

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Terms that contribute at $O(\alpha_S)$/NLO $\Rightarrow$ double counting.

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\text{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx\,\frac{O(x)R(x) - O(0)A(x)}{x}$$

$$\langle O\rangle_{\text{PS}} = BO(0)\left[1 - \int_\mu \frac{dx}{x}P(x)\right] + \int_\mu dx\,O(x)B\frac{P(x)}{x}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O \rangle_{\mathrm{NLO}}{}' = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x} + \int_\mu \frac{dx}{x} P(x) - \int_\mu dx\, O(x) B \frac{P(x)}{x}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\mathrm{NLO}}{}' = BO(0) + \bar{V}O(0) + \int_0^1 dx\,\frac{O(x)R(x) - O(0)A(x)}{x}$$
$$+ \int_\mu \frac{dx}{x}P(x) - \int_\mu dx\,O(x)B\frac{P(x)}{x}$$

Result ("MC@NLO master formula")

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B + \bar{V} + \int_0^1 dx\,\frac{P(x) - A(x)}{x}\right]$$
$$+ \int dx\,O(x)\frac{R(x) - P(x)}{x}\ .$$

Note: $(O(0)B \otimes$ parton shower) adds back subtracted terms
$\Rightarrow$ NLO result is exactly reproduced after parton shower.

Matching MC and NLO

$$\langle O \rangle_{\mathrm{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{P(x) - A(x)}{x} \right] + \int dx \, O(x) \frac{R(x) - P(x)}{x} \; .$$

Observations/remarks:

- Events with n and $n+1$ legs are seperately finite. No cancellation of large weights.
- NLO result can be recovered strictly upon expansion in powers of α (with parton shower emission).
- Interface to MC program very well defined.
- Dropping $\mu \to 0$ is only a power correction.

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B + \bar{V} + \int_0^1 dx\,\frac{P(x)-A(x)}{x}\right] + \int dx\,O(x)\frac{R(x)-P(x)}{x}\ .$$

Three types of matching

1. MC@NLO (classic, Frixione and Webber).
2. Simpler: parton shower with $P(x) = A(x)$.
3. Or, also simpler, $P(x) = R(x)$.

Matching MC and NLO

$$\langle O \rangle_{\text{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{P(x) - A(x)}{x} \right] + \int dx \, O(x) \frac{R(x) - P(x)}{x} \, .$$

1. Classic MC@NLO (Frixione and Webber)
   - $A(x) =$ FKS subtraction terms
   - $P(x)$ and phase space specific for HERWIG.
   - Generic, calculate once and for all.
     (Usually, $A(x)$ and $P(x)$ factorize off B.)
   - New for every process.

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{P(x)-A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x)-P(x)}{x}\;.$$

2. ‘Custom’ parton shower
e.g. with Catani–Seymour subtraction kernels

- CS subtraction already used in many NLO calculations.
- $P(x) = A(x)$, so terms vanish.
- $R(x) - A(x)$ already in NLO parton level program.

⇒ (almost) no need to modify NLO calculation!

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B + \bar{V} + \int_0^1 dx\,\frac{P(x)-A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x)-P(x)}{x}\ .$$

3. Simpler in a different way, $P(x) = R(x)$

- $R(x) - A(x)$ now only needed as integral available in NLO parton level program.
- No $n+1$ body events.
- ≥ 1 PS emission from $R(x)$ as splitting kernel $\rightarrow$POWHEG.
- Positive weights (terms $\neq 0$ are $\sigma^{\mathrm{incl}}_{\mathrm{NLO}}$).
- Further emissions from (truncated) standard PS.

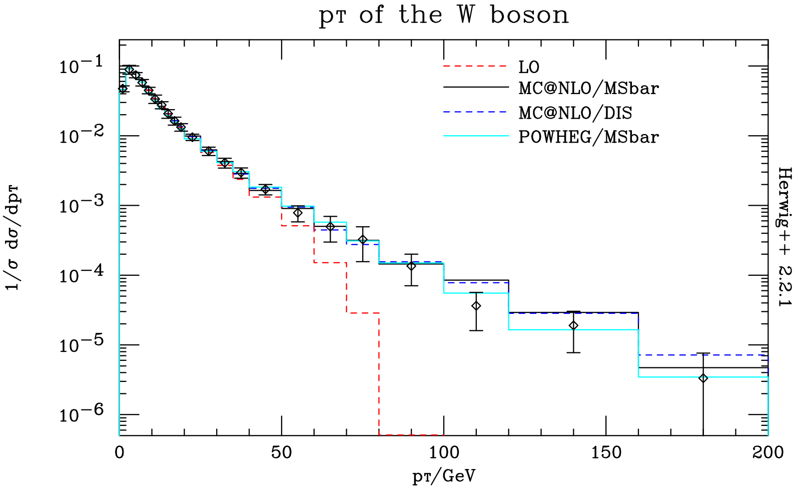
MC@NLO (in Herwig++)

- Introduced 2002 — Frixione, Webber, JHEP 0206:029,2002 [hep-ph/0204244].
- Extended to heavy quarks

  Frixione, Nason, Webber, JHEP 0308:007,2003 [hep-ph/0305252].
- further extensions to many processes (single top etc.)
- MC@NLO customised to use with HERWIG.
- Some processes in Herwig++ as well
  $e^+e^- \to$ jets, DY, W', h^0 decay

  Latunde–Dada 0708.4390, 0903.4135, Latunde–Dada, Papaefstatiou, 0901.3685.

Drell–Yan example



Latunde–Dada 0708.4390, 0903.4135, Latunde–Dada, Papaefstathiou, 0901.3685.

POWHEG

- Alternative proposed by P. Nason.
- Modified Sudakov FF for first emission.
- Angular ordered Parton Shower tricky (see below).
- *Truncated Shower* adds in missing radiation afterwards.
- Finally evolution with 'ordinary' Parton Shower.

[Nason, hep-ph/0409146; Nason, Ridolfi hep-ph/0606275]

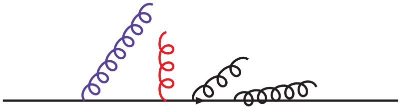
Recently systematically extended.

- `POWHEG` formulation independent of the event generator implementation.
- Worked out for different subtraction schemes.

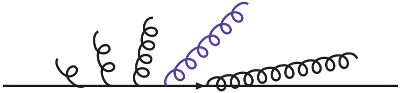
[Frixione, Nason, Ridolfi, 0707.3081, 0707.3088; Frixione, Nason, Oleari, 0709.2092]

POWHEG

Angular ordered showers and POWHEG



$p_\perp$ ordered shower. Angular ordering from additional vetos.



Angular ordered shower. Some softer emissions before hardest one.

Need truncated showers.

POWHEG in Herwig++

- First implementation of method for e^+e^- annihilation

  [O. Latunde–Dada, SG, B. Webber, hep-ph/0612281]

- Many more processes now available with release: DY $(\gamma^*/Z^0/W^\pm), h^0, h^0Z^0, h^0W^\pm$

  [K. Hamilton, P. Richardson and J. Tully, 0806.0290, 0903.4345]

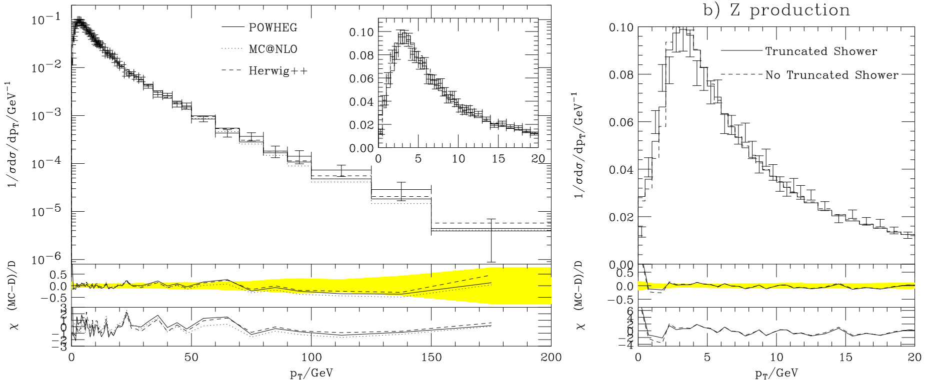
- and with contributed code: $e^+e^- \to \text{jets}, t\bar{t}, t-\text{decay}, W', h^0-\text{decay}$

  [O. Latunde-Dada, 0812.3297, Eur. Phys. J. C **58**, 543 (2008)]

  [A. Papaefstathiou and O. Latunde-Dada, JHEP **0907**, 044]

- includes full truncated showers.
- Interface to work of Nason *et.al.* straightforward.
- More processes underway.

POWHEG in Herwig++ with full truncated shower.
DY γ^*/Z^0 production vs CDF Run I.



[K. Hamilton, P. Richardson, J. Tully, 0806.0290]

Dipole showers

- (ARIADNE, of course. Not invented for NLO matching.)
- Idea to use CS subtraction terms for parton shower immediately clear from MC@NLO. Written up in

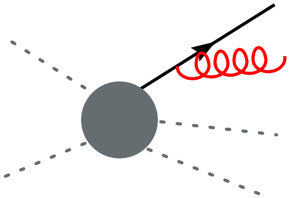
  [Nagy, Soper, JHEP 0510:024 (2005)]

- Catani–Seymour Dipole cascade codes recently implemented by Schumann, Krauss [0709.1027] (with CKKW like matching) and (independently) Dinsdale, Ternick, Weinzierl [0709.1026]
- Lund Dipoles revisited [J. C. Winter and F. Krauss, JHEP **0807**, 040 (2008)]
- Similar approach (VINCIA), based on Antenna subtraction by Giele, Kosower, Skands (toy process) [0707.3652]

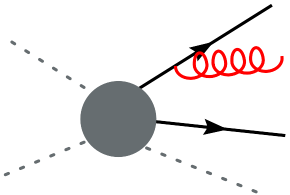
No NLO matching yet. Coherence/angular ordering not clear.

Why all those dipoles?

Momentum conservation in shower emissions.



$1 \to 2$ emissions. No momentum conservation. Kinematic reshuffling needed afterwards. Difficult for matching with NLO.



$2 \to 3$ emissions. Momentum conservation simple due to recoil against 3rd parton.

Coherent dipole shower

Our latest results [Simon Plätzer, SG, 0909.5593]

- Investigated parton showers with local recoils (that's what a CS parton shower is like).
- Angular ordering/soft coherence manifest in Sudakov anomalous dimension

$$\Gamma_q(p_\perp^2, Q^2) = C_F \left(\ln \frac{Q^2}{p_\perp^2} - \frac{3}{2} \right) ,$$

$$\Gamma_g(p_\perp^2, Q^2) = C_A \left(\ln \frac{Q^2}{p_\perp^2} - \frac{11}{6} \right) .$$

[A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rept. 100 (1983) 201]

[G. Marchesini, B.R. Webber, NPB 238 (1984) 1]

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[A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rept. 100 (1983) 201]

[G. Marchesini, B.R. Webber, NPB 238 (1984) 1]

- Not correct for virtuality ordered showers.

Result for Herwig++ like shower (ordering in $\tilde{q}$) with recoils

$$\Gamma_q^{AO}(p_\perp^2, Q^2) = C_F\left(\ln\frac{Q^2}{p_\perp^2} - \frac{3}{2}\right) + C_F\frac{p_\perp}{Q}\left(1 - 2\lambda\frac{Q^2}{s_{ik}}\right) + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right),$$

$$\Gamma_g^{AO}(p_\perp^2, Q^2) = C_A\left(\ln\frac{Q^2}{p_\perp^2} - \frac{11}{6}\right) + 2C_A\frac{p_\perp}{Q}\left(1 - \lambda\frac{Q^2}{s_{ik}}\right) + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right).$$

Recoil $\rightarrow$ power correction, beyond NLL.
(Only enters in phase space ($\rightarrow \lambda$) as spectator is only rescaled.)

[Simon Plätzer, SG, 0909.5593]

Coherent dipole showers

Result for CS dipole shower with $p_\perp$ ordering and right choice of phase space boundaries.

$$\Gamma_q^{CS}(p_\perp^2, \cdot) = C_F\left(\ln\frac{s_{ik}}{p_\perp^2} - \frac{3}{2}\right) - C_F\pi\lambda\frac{p_\perp}{\sqrt{s_{ik}}} + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right),$$

$$\Gamma_g^{CS}(p_\perp^2, \cdot) = C_A\left(\ln\frac{s_{ik}}{p_\perp^2} - \frac{11}{6}\right) - C_A\pi\lambda\frac{p_\perp}{\sqrt{s_{ik}}} + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right).$$

Hard scale $Q^2 = s_{ik}$ = dipole inv mass.
Recoil → power correction, beyond NLL.
$p_\perp$ ordering → simple POWHEG matching.

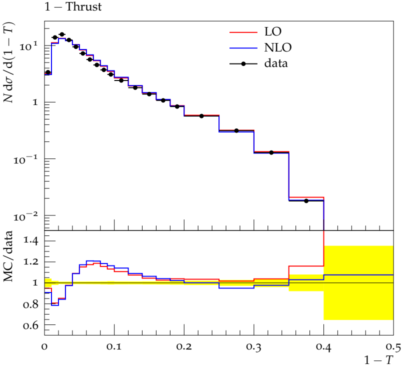
[Simon Plätzer, SG, 0909.5593]

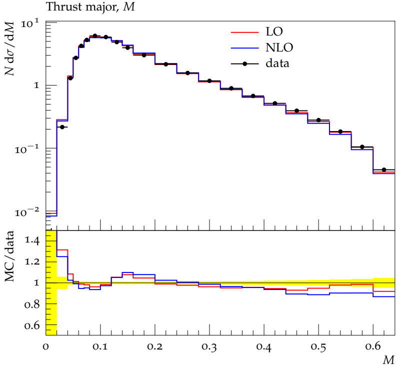
New coherent dipole shower

Code in testing phase. [Simon Plätzer, SG, in preparation]

- Integrates with Herwig++ as alternative shower module.
- Coherence (see above).
- Recoils build up from multiple emissions in the IS.
- e^+e^- tested/tuned on hadron level.
- Drell Yan tested on parton level.
- DIS and jet hadroproduction to be tested further.

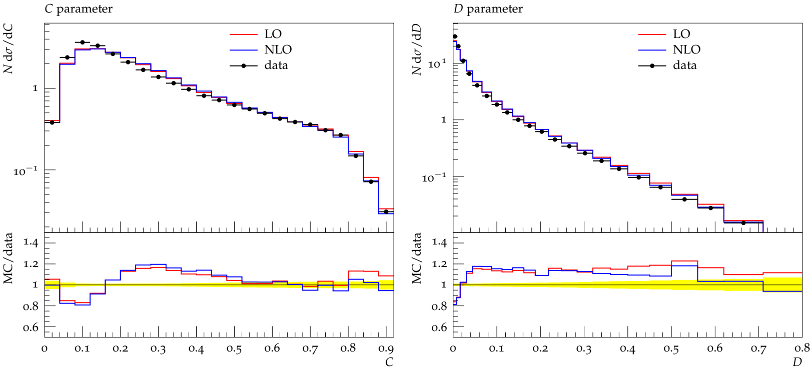
Integrated into Herwig++. Preliminary tune to LEP data.
LO and NLO matched (POWHEG like).





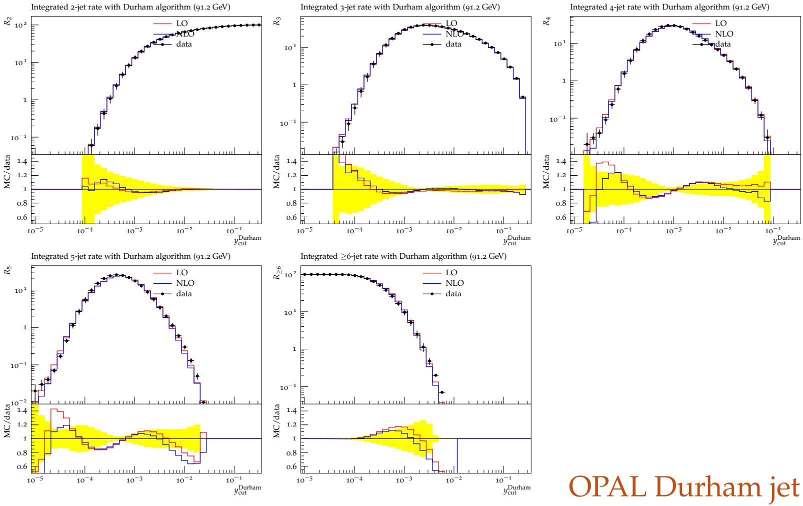
Thrust, Thrust major.

Integrated into Herwig++. Preliminary tune to LEP data.
LO and NLO matched (POWHEG like).



C, D Parameter

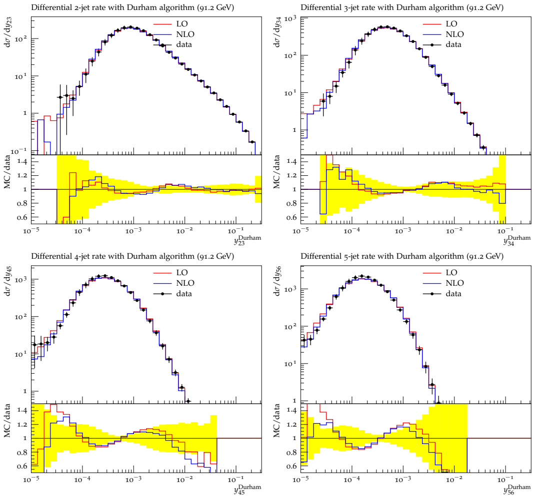
Integrated into Herwig++. Preliminary tune to LEP data.
LO and NLO matched (POWHEG like).



OPAL Durham jet rates

e^+e^-

Integrated into Herwig++. Preliminary tune to LEP data.
LO and NLO matched (POWHEG like).



Durham Y_{nm}.

- Relevant processes checked.
- Matching machinery very generic.
- Easy to adopt CS subtracted NLO code.
  (that was the idea…)
- MC@NLO and POWHEG matching implemented.
- Integrates well into Herwig++.
- Next: check more complicated process to see how generic this really is…
- Stay tuned…

Conclusions

- New C++ codes work very well.
- Become standard for LHC era.
- A lot of physics in Herwig++.

- NLO matching has matured.
  Close to standard requirement for important processes.
- New coherent dipole shower.
- Road towards simple matching of
  NLO codes and MC.