NNLO Higgs production via gluon fusion with finite top mass

Kemal Ozeren

Bergische Universität Wuppertal

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(work done in collaboration with Robert Harlander)
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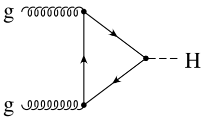
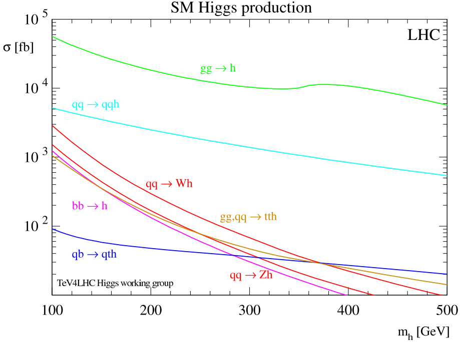
arXiv:0909.3420

Outline

$gg \to H$ in the SM

- production is via a top loop





- LO cross section known

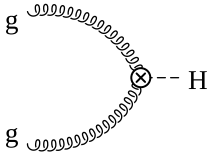
$$\sigma_{LO} = \frac{G_F \alpha_s^2(\mu^2)}{128\sqrt{2}\pi} \tau^2 \, \delta(1-x) |1 + (1-\tau) f(\tau)|^2$$

$$\tau = \frac{4m_t^2}{m_H^2}$$

[Georgi, Glashow, Machacek, Nanopoulos]

The Heavy Top Effective Theory

- If $\frac{m_H}{2m_{top}} \ll 1$ work in effective theory
- Top 'integrated out' of the theory
- ...but leaves its legacy in the form of altered couplings and new vertices



$$\mathcal{L}_{eff} = -\frac{H}{4v} C_1 G_{\mu\nu} G^{\mu\nu}$$

$$C_1 = -\frac{1}{3}\frac{\alpha_s}{\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \cdots\right\}$$

Major benefit: reduces number of loops by one

Quantum Corrections

- QCD corrections huge - $\mathcal{O}(100\%)$

  NLO (effective theory) [Dawson'91]

  NLO (HIGLU) [Spira,Djouadi,Graudenz,Zerwas'95]

  NNLO (effective theory) [Harlander,Kilgore'02]
  [Anastasiou,Melnikov'02]
  [Ravindran,Smith,van Neerven'03]

- Electroweak
  [Actis, Passarino, Sturm, Uccirati '08]

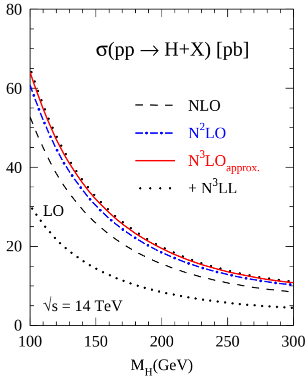
- Mixed QCD-Electoweak
  [Anastasiou, Boughezal, Petriello '08]

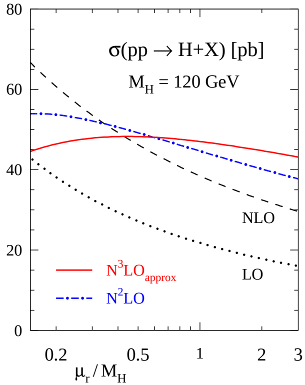
- NNLO+NNLL - $\mathcal{O}(\%)$
  [Catani, de Florian, Grazzini, Nason '03]

- N^3LO threshold enhanced corrections
  [Moch, Vogt '05], [Laenen, Magnea '05], [Ravindran '05]
  [Kidonakis '05], [Idilbi, Ju, Yuan '05]

- "π^2-resummation"
  [Ahrens, Becher, Neubert, Yang '08]

Overview of QCD Corrections





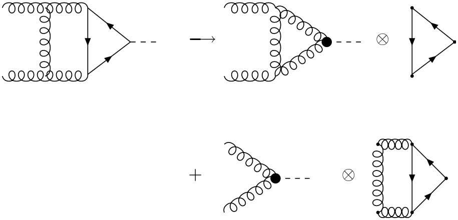
Moch, Vogt

- QCD effects well under control
- Residual scale uncertainty $\sim 5\%$
- See also updated analyses
  [Anastasiou, Boughezal, Petriello ’08]
  [de Florian, Grazzini ’09]

How accurate is the effective theory at NNLO?

Asymptotic Expansion

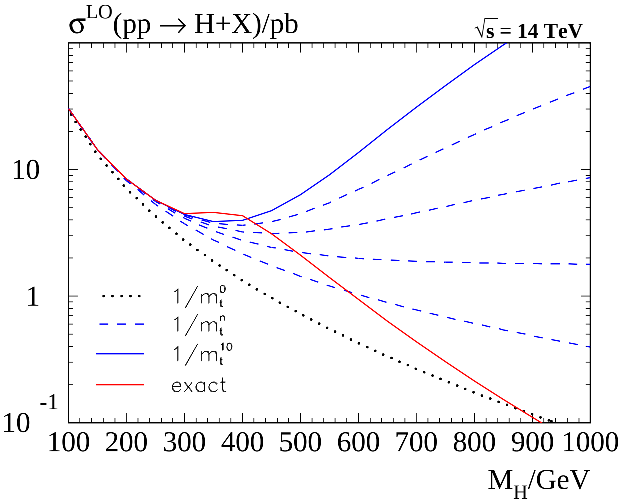
- Full NNLO calculation with top mass not currently feasible
- We perform an asymptotic expansion in $\frac{1}{m_t}$



$$\sigma = \sum_n \left(\frac{m_H^2}{4m_t^2} \right)^n \sigma_n$$

- First term σ_0 is the effective theory result
- First non-leading $1/m_t$ term at NLO known
  `[Dawson,Kauffman '93]`
- Tools exist to automatize the calculation
  `QGRAF, EXP, FORM, MATAD, MINCER`

Expansion of σ_{LO}



Small x behaviour

The total cross section is

$$\sigma = \int_{x_{min}}^{1} \mathcal{L}(x)\,\hat{\sigma}(x)\,dx$$

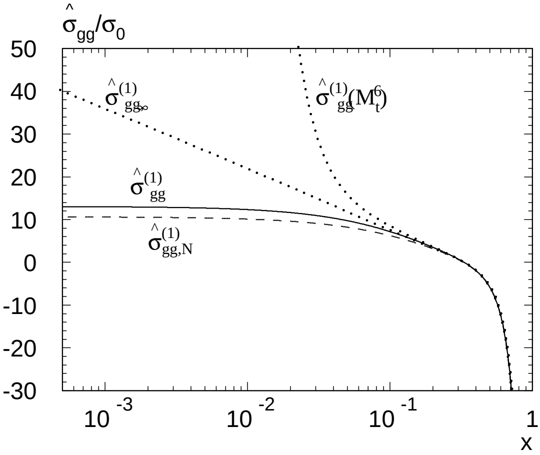
- When $x = m_H^2/\hat{s} \to 0$ our approach breaks down
- But: the leading small-x behaviour is known exactly:
  [Marzani,Ball,Del Duca,Forte,Vicini '08]

$$\begin{aligned}\hat{\sigma}_{gg}^{(1)} &= 3\,\sigma_0\,\mathcal{C}^{(1)} + \mathcal{O}(x)\\ \hat{\sigma}_{gg}^{(2)} &= -9\,\sigma_0\,\mathcal{C}^{(2)}\,\ln x + c + \mathcal{O}(x)\end{aligned}$$

We can therefore improve our result. For example, at NLO we use

$$\hat{\sigma}_{gg}^{(1)}(x) = \hat{\sigma}_{gg,N}^{(1)}(x) + (1-x)^{N+1}\left[3\,\sigma_0\mathcal{C}^{(1)} - \hat{\sigma}_{gg,N}^{(1)}(0)\right]$$

NLO Results



- Both effective theory result and our expansion have spurious small x behaviour
- We improve it by incorperating the known exact $x \to 0$ piece

NLO $1/m_t$ expansion - comparison with HIGLU

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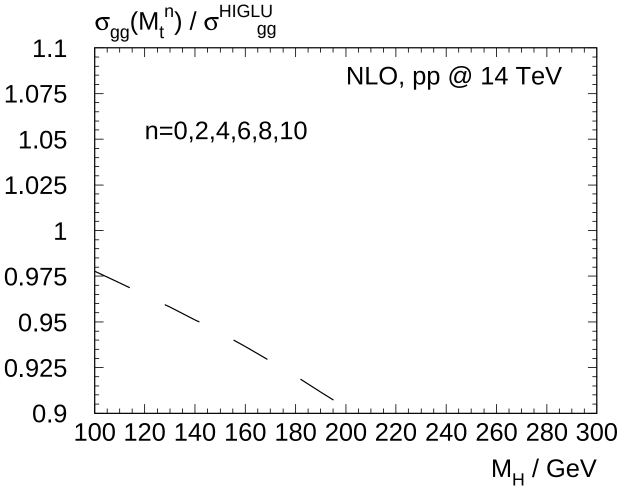

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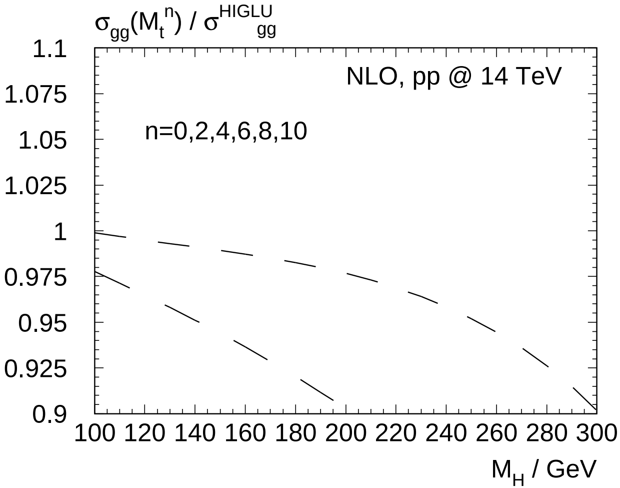

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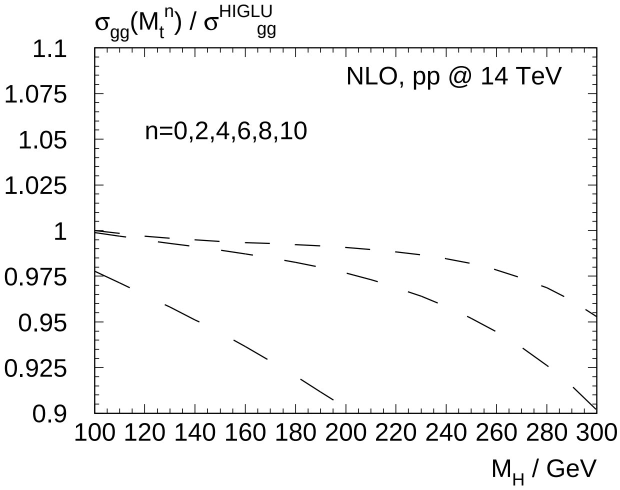

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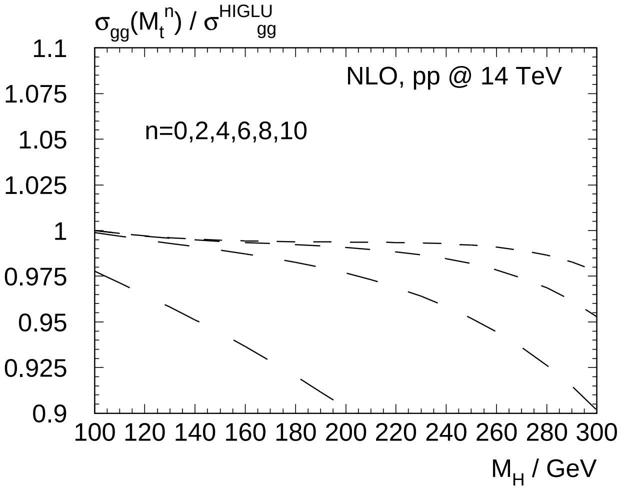

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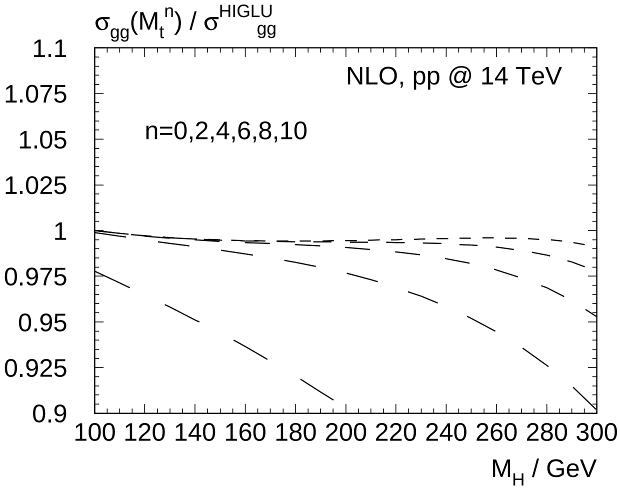

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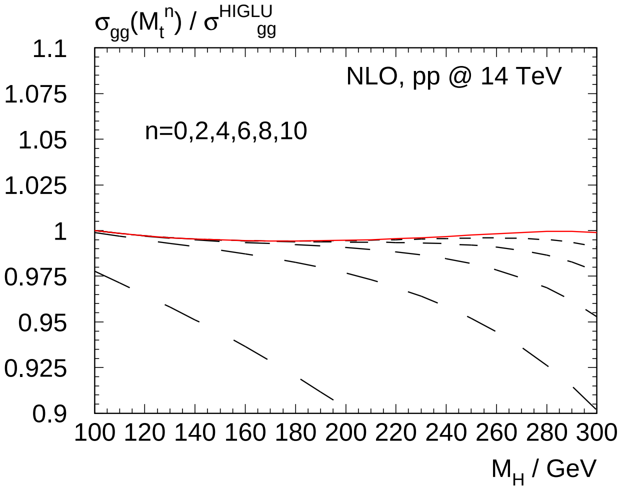

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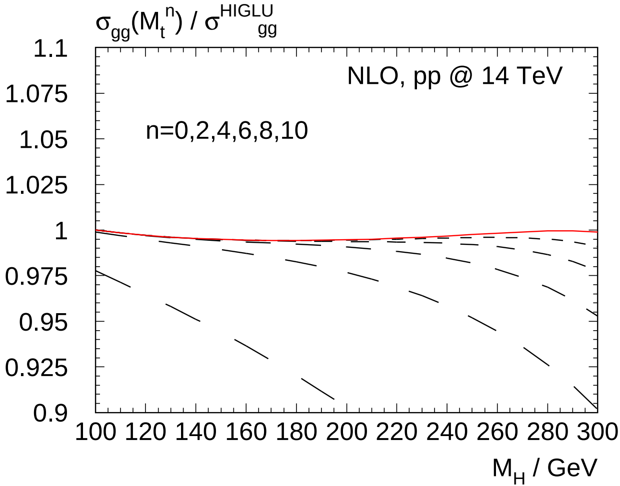


- Excellent agreement
- Use the same approach at NNLO

NNLO ingredients

We have three contributions:

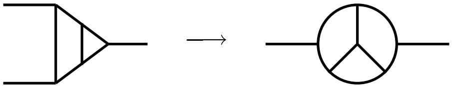
- double virtual
  [Harlander,KJO ’09]
  [Pak,Rogal,Steinhauser ’09]
- single real emission
- double real emission

$$
\begin{aligned}
\sigma &= \int M^{(3)}_{gg\to H}\left[M^{(1)}_{gg\to H}\right]^* + \left|M^{(2)}_{gg\to H}\right|^2 \\
&+ \iint M^{(2)}_{gg\to Hg}\ \left[M^{(1)}_{gg\to Hg}\right]^* \\
&+ \iiint \left|M^{(1)}_{gg\to Hgg}\right|^2
\end{aligned}
$$

- Each integrated over relevant phase space volume

Loop Amplitudes

- After expansion, double virtual part consists of 2-loop 3-point diagrams
- Use Baikov-Smirnov method to map onto known 3-loop 2-point diagrams
  [hep-ph/0001192]



- First used to evaluate the virtual contribution to the Higgs cross section in the effective theory
  [Harlander '00]
- Can treat arbitrary propagator powers

Phase Space Integration (1)

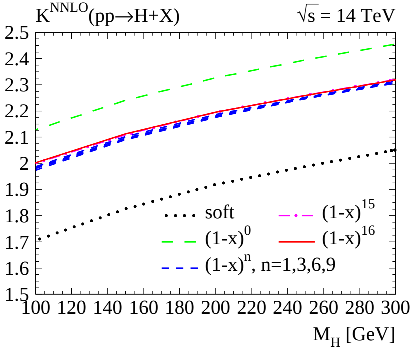
- One particle final states are easy $\longrightarrow \delta(1 - \frac{m_H^2}{\hat{s}})$
- Two particle final states are somewhat less easy
  - Amplitudes come with ${}_2F_1$ hypergeometrics
  - Direct integration yields extended hypergeometrics $({}_3F_2,\ {}_4F_3)$

$$\begin{aligned} & \int_0^1 dv\, v^\alpha (1-v)^\beta\, {}_2F_1(1, -\epsilon, 1-\epsilon, z\, v) \\ = \; & B(\alpha,\beta)\, {}_3F_2\left[\{1,-\epsilon,1+\alpha\},\{1-\epsilon,\alpha+\beta+2\},z\right] \end{aligned}$$

- Use HypExp package to expand these in ϵ
  [Huber,Maitre]

Phase Space Integration (2)

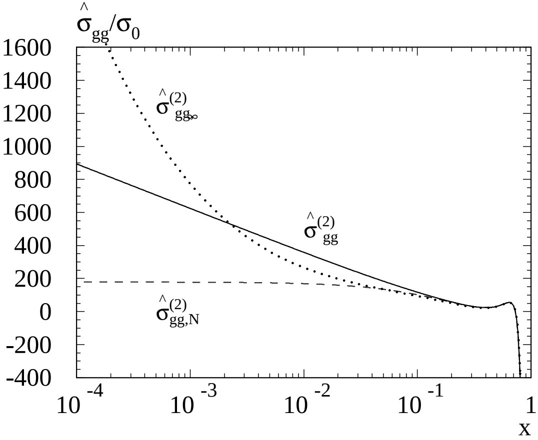
- Three particle final states very difficult
- Expand amplitude and phase space in powers of $(1 - \frac{m_H^2}{\hat{s}})$
- Series converges quickly



[Harlander,Kilgore]

- In order to cancel poles must also expand the single real contribution

Results at NNLO



- Effective theory result has spurious high energy behaviour
- We match our result onto the exact small x result (solid curve)

NNLO $1/m_t$ expansion - comparison with effective theory

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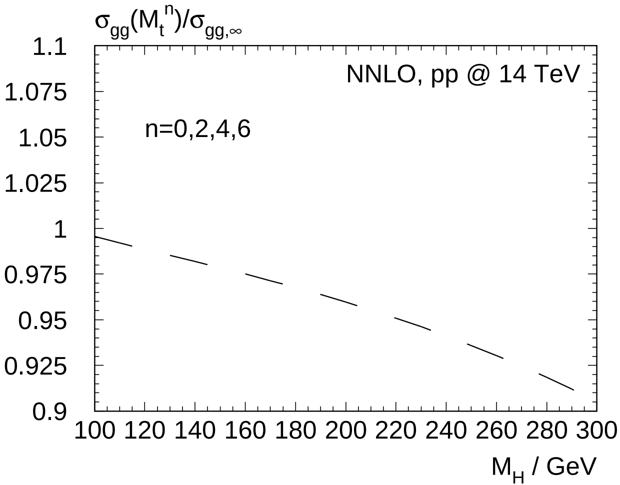

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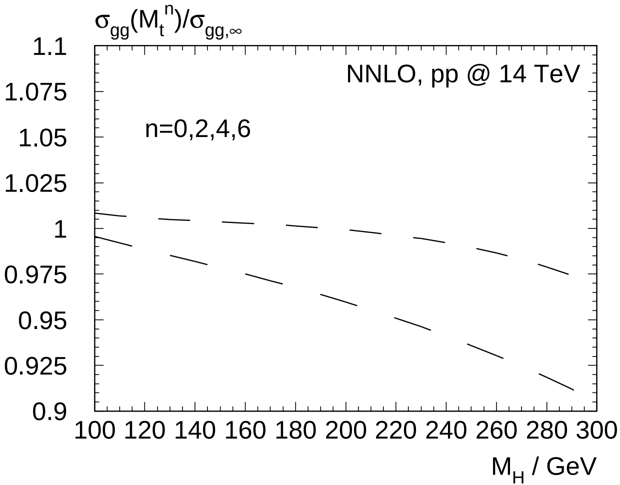

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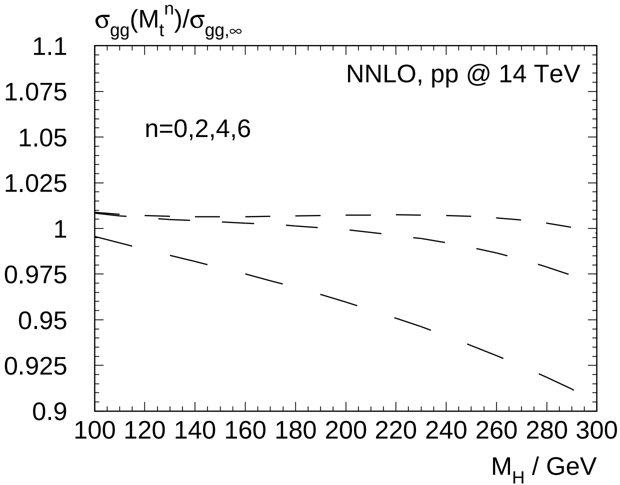

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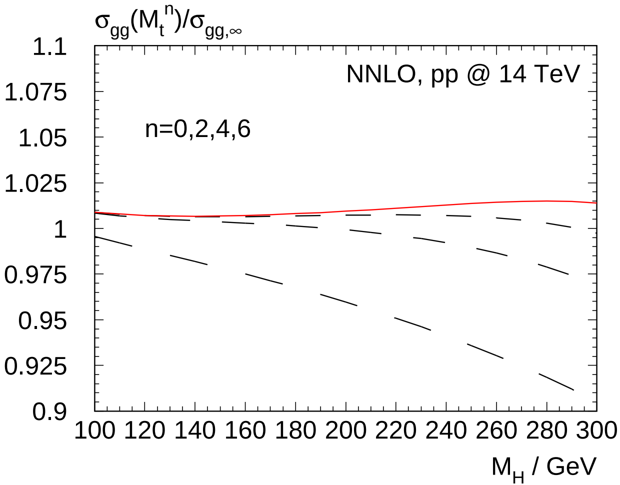

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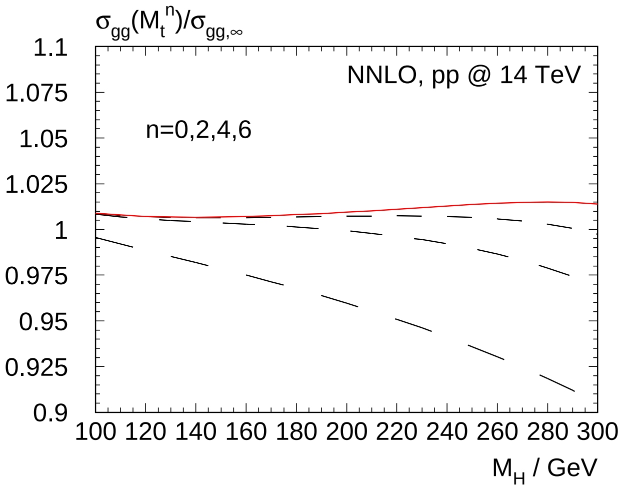

- Small deviations, $\simeq 1\%$

Summary and Outlook

- Long-standing problem: how accurate is the large m_t limit at NNLO?
- We have shown that top mass effects are small - about 1%
- Use of effective theory is justified

Next Steps:

- Consider effects on exclusive quantities