AutoDipole
: an Automated Dipole Subtraction

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Reference: Nucl.Phys.Proc.Suppl.183:268-273,2008
arXiv:0807.3701 [hep-ph]

Long write up will appear soon

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1. Introduction

- Large Hadron Collider (LHC) at CERN

  - Energy Frontier : $\sqrt{S} \simeq 14\text{TeV}$

    $\longrightarrow$ Direct production of Higgs and new particles beyond the Standard Model

  - Proton-Proton collision : $pp \to \text{X}$

    $\longrightarrow$ Triggered by the QCD interaction

  - New physics identification requires the Standard Model predictions

$$(\text{New Physics}) = (\text{LHC signals}) - (\text{the SM predictions})$$

  - The rate of QCD processes with high momentum transfer can be predicted by the perturbative expansion in the small strong coupling constant

    For example, $\alpha_s(m_t) \simeq 0.1$

■ Perturbative QCD

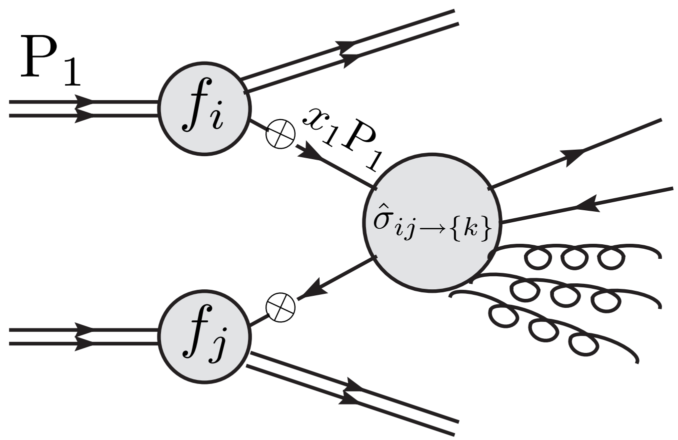
- Master Formula : Factorization of the hard scattering process

$$\sigma_{pp\to X} = \sum_{i,j,\{k\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij\to\{k\}}(\alpha_s, Q) \otimes D_{\{k\}\to X}$$

Parton distribution function
(Non-perturbative)

Subprocess partonic cross section
(Perturbative)

Jet algorithm
Parton shower
Hadronization model



- Perturbative expansion of the partonic cross section

$$\hat{\sigma}_{ij\to\{k\}} = \sigma_{\mathrm{LO}}(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \cdots)$$

Leading order (LO)
Next-to-leading order (NLO)
Next-to-next-to-leading order (NNLO)

■ Leading order (LO)

- LO(Tree level) is well automated

Alpgen, CompHep, CalcHEP, FeynArts/FeynCalc, GRACE, HELAC/PHEGAS, MadGraph, $\cdots$

■ Next-to-leading order (NLO)

- LO has a large uncertainty from the renormalization/factorization scale dependences
- NLO is not yet fully automatized
- Process with multi-parton legs are difficult
- LHC priority NLO wish list in Les Houches 2005 (hep-ph/0604120)

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

-A. Bredenstein, A.Denner, S.Dittmaier, S.Pozzorini
arXiv:0807.1248 0905.0110

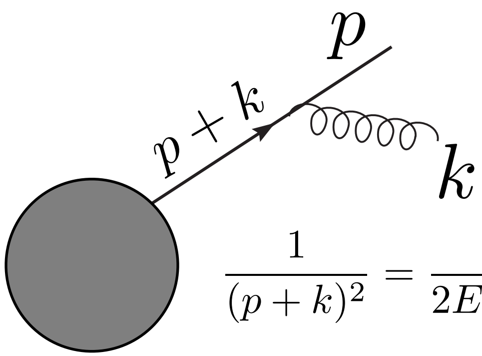
- R. Keith Ellis, Kirill Melnikov, Giulia Zanderighi
0901.4101 0906.1445

- C.Berger, Z.Bern, L.Dixon, F.Cordero, D.Forde, T.Gleisberg, H.Ita, D.Kosower, D.Maitre
0902.2760 0907.1984

-These predictions are urgently needed for the successful operation of LHC

-The computation of these radiative corrections is now a very active field

- QCD at NLO : $\sigma_{\rm NLO} = \sigma_{\rm real} + \sigma_{\rm virtual}$
- Real correction
  - Soft and collinear singularities



$$\frac{1}{(p+k)^2} = \frac{1}{2E_k E_p(1-\beta\cos\theta_{kp})} \longrightarrow \infty$$

$$\begin{cases} \text{Soft limit } \ \mathrm{E}_k \ \rightarrow \ 0 & \text{: Soft singularity} \\ \text{Collinear limit } \ \theta_{kq} \rightarrow 0 & \text{: Collinear singularity} \end{cases}$$

- Cancellation of soft/collinear singularities

$$\sigma_{\rm NLO} = \sigma_{\rm real} + \sigma_{\rm virtual}$$

$\sigma_{\rm real}$ → Soft/Collinear

$\sigma_{\rm virtual}$ → Soft/Collinear

$\sigma_{\rm virtual}$ → UV → Renormarization → Finite

Soft/Collinear + Soft/Collinear → Finite in observables :IR safe (more precisely, soft/collinear safe)

Kinoshita-Lee-Nauenberg theorem

Dipole subtraction

- A general and practical procedure to treat soft/collinear singularities at QCD NLO

S.Catani and M.H.Seymour, Nucl.Phys.B485(1997)291
S.Catani, S.Dittmaier, M.H.Seymour, Z.Trocsanyi, Nucl.Phys.B627(2002)189

1. Construct the counter terms which cancel all soft/collinear singularities
2. Subtract it from σ_{real} and add it to $\sigma_{\mathrm{virtual}}$

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

$$= (\sigma_{\mathrm{real}} - \sigma_{\mathrm{a}}) + (\sigma_{\mathrm{virtual}} + \sigma_{\mathrm{a}})$$

$$= \underbrace{\int d\Phi_{m+1} \left[|\mathrm{M}_{\mathrm{real}}|^2 - \sum_i \mathrm{D}_i \right] \Bigg|_{\mathrm{D}=4}}_{\text{Finite}} + \underbrace{\int d\Phi_m \left[|\mathrm{M}_{\text{1-loop}}|^2 + \underline{\int d\Phi_1 \sum_i \mathrm{D}_i} \right] \Bigg|_{\mathrm{D}=4}}_{\text{Finite}}$$

$= \mathrm{I}$ -term

- Real correction: Instead of reguralizing, subtract
  Calculation (phase space and matrix element) is in 4-dimension

- Dipole term is systematically constructed based on the factorization of soft/collinear singularities $\longrightarrow$ reduction to Born level

$$\mathrm{D}_i \simeq \frac{1}{s_i} \mathrm{V}_i \cdot |\mathrm{M}_i|^2_{\mathrm{Born}}$$

Singular part

dipole splitting function (Universal)

- Integration of dipole term is analitically done once for all

■ Multi-parton leg processes

- Dipole subtraction makes it possible
- It requires many dipole terms and repeats the same kinds of calculation at huge times (Order 100 dipoles)
- The algorithm is a combinatorial way

⇨ The automatization is required and it is possible

■ Our aim

1. Automatize the dipole subtraction
2. Apply it to the QCD backgrounds and the relevant signals in LHC

-There is recent work in the same direction

- T. Gleisberg and F. Krauss, Eur.Phys.J.C53(2008)501, arXiv0709.2881
- M.H. Seymour and C. Tevlin, arXiv0803.2231
- R. Frederix and T. Gehrmann and N. Greiner, JHEP0809:122, arXiv0808.2128
- M. Czakon, C.G. Papadopoulos, M. Worek, JHEP0908:085, arXiv0905.0883

2. AutoDipole package

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}} + \sigma_{\mathrm{coll}}$$

$$= \int d\Phi_{n+1} \left(|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i \right) + \int d\Phi_n \left(|\mathrm{M}|^2_{\text{1-loop}} + \sum_i \mathrm{I}_i \right) + \int dx \int d\Phi_n \sum_i (\mathrm{P}_i + \mathrm{K}_i)$$

- Mathematica code and an interface with MadGraph

- Automatic creation of all D, I, P, and K terms
  - D term: Version 1.0beta publicly available at wep page,

    Computer Algebra and Particle Physics 2009, DESY Zeuthen
    //https://indico.desy.de/conferenceOtherViews.py?view=standard&confId=1573

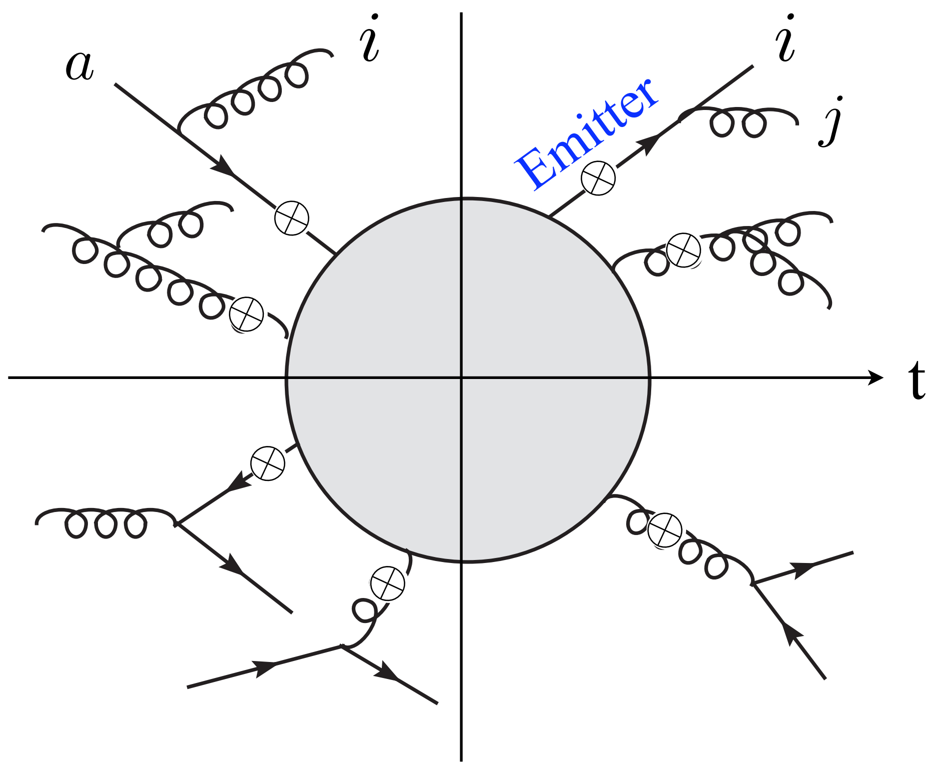
  - I term: Completed and publicly available soon

  - P and K terms: will be completed and publicly available soon

2-1 Dipole term

■ Algorithm and formulae

1. Choose emitter pair



Choose all possible leg-pair which matches one of the seven patterns

(a, i) or (i, j)

Initial parton=a,b
Final parton=i,j,k

2. Choose spectator

Choose a different leg from emitter pair

Spectator : $k \neq i, j$ $\quad b \neq a$

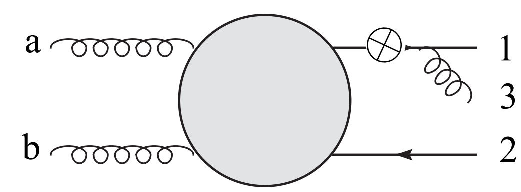
spectator / emitter pair	k	b
(i, j)	$\mathrm{D}_{ij,k}$ $(k \neq i, j)$	$\mathrm{D}_{ij}^{\ b}$
(a, i)	$\mathrm{D}^{ai}_{\ k}$ $(k \neq i)$	$\mathrm{D}^{ai,b}$ $(b \neq a)$

3. Use dipole formulae

$$\mathrm{D}_{ij,k}(p_1,\cdots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} < 1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1|\frac{\mathrm{T}_k\cdot\mathrm{T}_{ij}}{\mathrm{T}_{ij}^2}\mathrm{V}_{ij,k}|1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1>_m$$

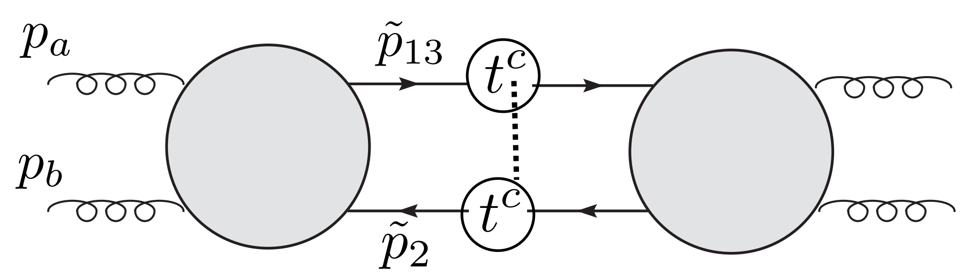
Example : $g(a)g(b)\rightarrow u(1)\bar{u}(2)g(3)$

$$\mathrm{D}_{13,2}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_1\cdot p_3}\langle gg\rightarrow\tilde{u}\tilde{\bar{u}}|\frac{\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}}{\mathrm{T}_{ug}^2}\mathrm{V}_{13,2}|gg\rightarrow\tilde{u}\tilde{\bar{u}}\rangle_2$$



- Dipole splitting function : $\mathrm{V}_{13,2}(z,y) = \delta_{ss'}8\pi\alpha C_F\left[\frac{2}{1-z_i(1-y_{ij,k})}-(1+z_i)\right]$

- Color linked Born squared (CLBS): $\langle gg\rightarrow\tilde{u}\tilde{\bar{u}}|\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}|gg\rightarrow\tilde{u}\tilde{\bar{u}}\rangle_2$



$$\mathrm{T}_{\mathrm{X}}^a = \begin{cases} t^a & (\mathrm{X} = \text{quark}) \\ f^a & (\mathrm{X} = \text{gluon}) \end{cases}$$

- Reduced momenta satisfy the energy-momentum conservation and on-shell condition

$$p_a^\mu + p_b^\mu = \tilde{p}_{13}^\mu + \tilde{p}_2^\mu \qquad \tilde{p}_{13}^2 = \tilde{p}_2^2 = 0$$

⇨ Make it possible to reduce into the physical born amplitude, which can be calculated by the well automated LO softwares

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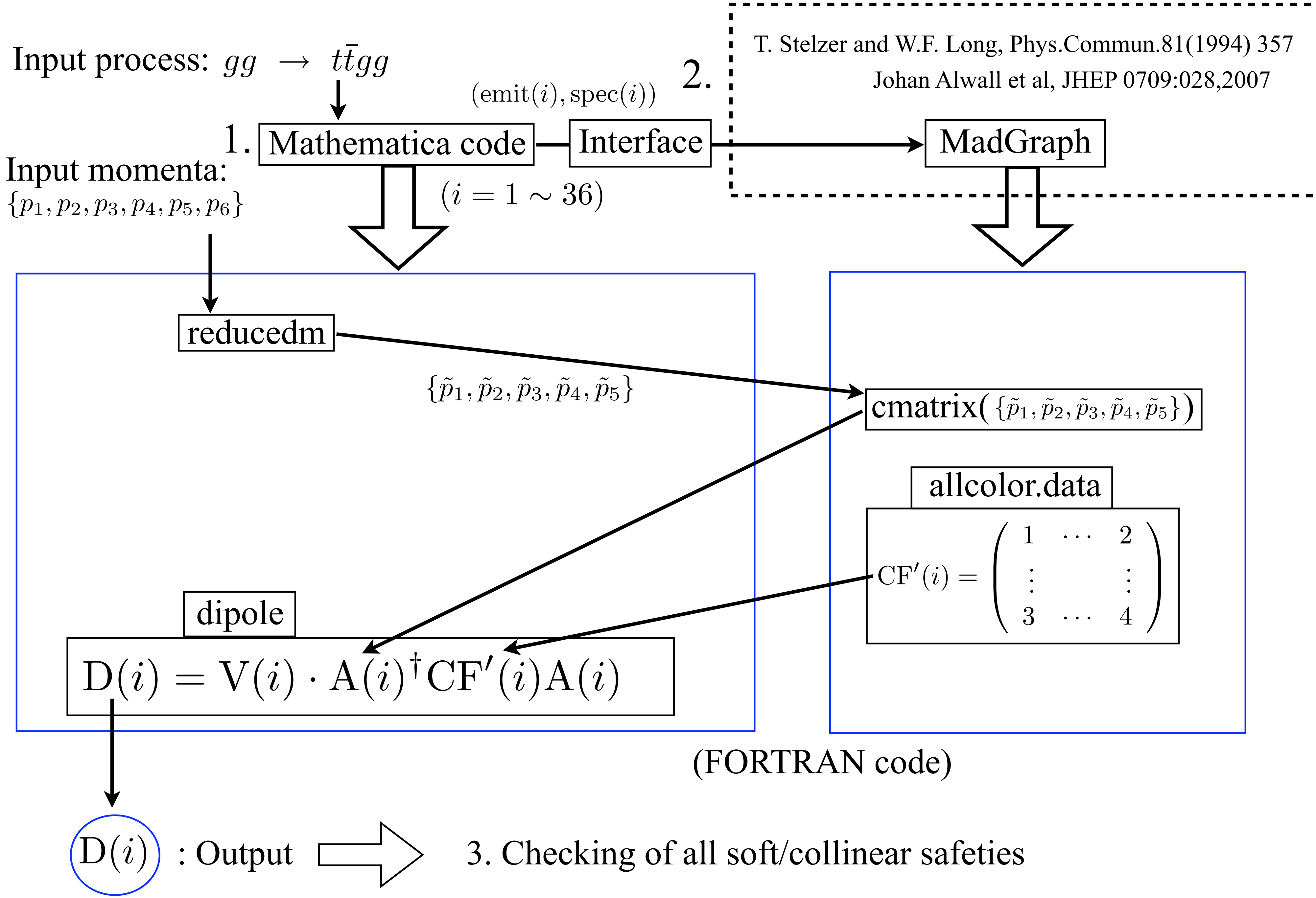
$$z_i = \frac{p_i\cdot p_k}{p_j\cdot p_k + p_i\cdot p_k} \qquad y_{ij,k} = \frac{p_i\cdot p_j}{p_i\cdot p_j + p_j\cdot p_k + p_k\cdot p_i} \qquad \tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}p_k^\mu \qquad \tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}p_k^\mu$$

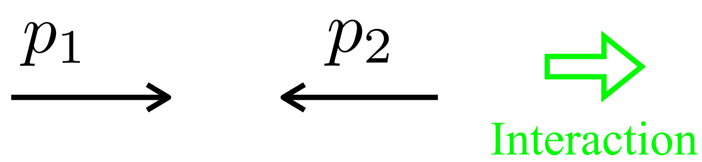
■ Scheme to calculate $|\mathrm{M}|^2 - \sum_i \mathrm{D}_i$ in AutoDipole

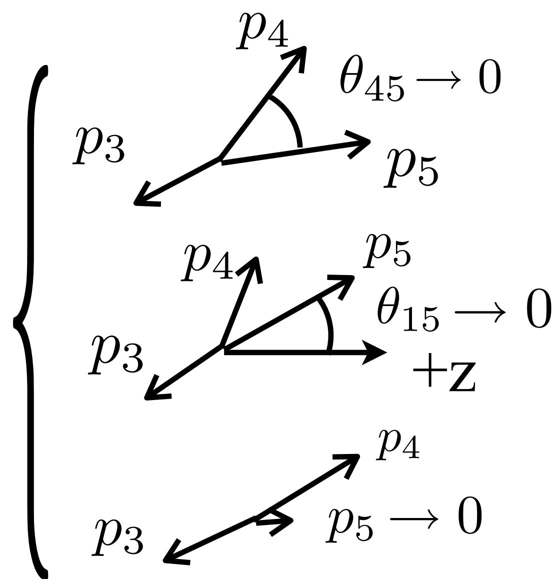


■ Check the soft/collinear limits

- Configurate all soft/collinear limits

$g(1)g(2) \to u(3)\bar{u}(4)g(5)$





: Colllinear limit of Final-Final spliting

: Colllinear limit of Initial-Final spliting

: Soft limit

	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
Massless				
(Including lepton)	$e^+e^- \to u\bar{u}g$			
	$e^-u \to e^-ug$			
	$e^-g \to e^-u\bar{u}$			
	$u\bar{u} \to e^+e^-g$			
(Parton only)	$gg \to u\bar{u}g$	$gg \to u\bar{u}gg$	$u\bar{u} \to d\bar{d}ggg$	
	$gg \to 3g$	$gg \to 4g$		
	$u\bar{u}g \to d\bar{d}g$			
(Including W/Z boson)		$\bar{u}u \to W^+W^-gg$	$gg \to W^+\bar{u}dgg$	
Massive				
(Including lepton)	$e^+e^- \to t\bar{t}g$			
(Parton only)	$gg \to t\bar{t}g$	$gg \to t\bar{t}gg$	$gg \to t\bar{t}ggg$	$u\bar{u} \to t\bar{t}b\bar{b}gg$
		$u\bar{u} \to t\bar{t}gg$	$gg \to t\bar{t}b\bar{b}g$	
		$ug \to t\bar{t}ug$		
		$\bar{u}g \to t\bar{t}\bar{u}g$		
		$gg \to t\bar{t}u\bar{u}$		
		$u\bar{u} \to t\bar{t}u\bar{u}$		

- Complete agreement with the dipole terms for $pp \to t\bar{t} + 1\text{jet}$

S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C59 (2009) 625 and
Phys.Rev.Lett.98(2007)262002

$$b_0 = |\bar{\mathrm{M}}|^2$$

$$d_0 = \sum_{all} \mathrm{D}(i)$$

	$b_0\,[\mathrm{GeV}^{-4}]$	$d_0\,[\mathrm{GeV}^{-4}]$
$g(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})g(p_c)g(p_d)$		
AutoDipole	$7.82039670869613 \cdot 10^{-10}$	$1.02594003852407 \cdot 10^{-9}$
Version 2	$7.82039670869605 \cdot 10^{-10}$	$1.02594003852405 \cdot 10^{-9}$
$q(p_a)\bar{q}(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})g(p_c)g(p_d)$		
AutoDipole	$1.12077211361620 \cdot 10^{-10}$	$1.22619016939900 \cdot 10^{-10}$
Version 2	$1.12077211361619 \cdot 10^{-10}$	$1.22619016939908 \cdot 10^{-10}$
$q(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})g(p_c)q(p_d)$		
AutoDipole	$2.75641273146785 \cdot 10^{-11}$	$4.79768338384667 \cdot 10^{-11}$
Version 2	$2.75641273146783 \cdot 10^{-11}$	$4.79768338384750 \cdot 10^{-11}$
$\bar{q}(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})\bar{q}(p_c)g(p_d)$		
AutoDipole	$3.46150168295956 \cdot 10^{-11}$	$8.34555795894942 \cdot 10^{-11}$
Version 2	$3.46150168295953 \cdot 10^{-11}$	$8.34555795894950 \cdot 10^{-11}$
$g(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})\bar{q}(p_c)q(p_d)$		
AutoDipole	$1.21420520114780 \cdot 10^{-11}$	$2.13553289076589 \cdot 10^{-11}$
Version 2	$1.21420520114779 \cdot 10^{-11}$	$2.13553289076600 \cdot 10^{-11}$
$q(p_a)\bar{q}(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})\bar{q}(p_c)q(p_d)$		
AutoDipole	$5.13710959990068 \cdot 10^{-12}$	$9.06330902408356 \cdot 10^{-12}$
Version 2	$5.13710959990063 \cdot 10^{-12}$	$9.06330902408350 \cdot 10^{-12}$
$u(p_a)\bar{u}(p_b) \to W^+(p_{w^+})W^-(p_{w^-})g(p_c)g(p_d)$		
AutoDipole	$0.627402537098012 \cdot 10^{-9}$	$0.114149934878320 \cdot 10^{-8}$
Version 2	$0.627402537098007 \cdot 10^{-9}$	$0.114149934878319 \cdot 10^{-8}$

- Complete agreement with the result for $u\bar{u} \to W^+W^-gg$ of $pp \to W^+W^- + 1\text{jet}$

S. Dittmaier, S. Kallweit, P. Uwer, Phys.Rev.Lett.100(2008)062003

2-2 I term

- Algorithm and formulae

1. Choose one emitter: i (in final) or a (in initial)
2. Choose one spectator: k (in final) or b (in initial) $\neq$ the choosed emitter in step 1
3. Use the corresponding formula

-For example, Emitter = massless quark, i
Spectator = massless quark or gluon, k

$$\mathrm{I}_f(i,k) = -\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} < \mathrm{T}_i \cdot \mathrm{T}_k > \frac{1}{C_F}\mathcal{V}_f(\epsilon)\left(\frac{\mu^2}{s_{ik}}\right)^\epsilon$$

$$\begin{cases} < \mathrm{T}_i \cdot \mathrm{T}_k > \ : \text{CLBS} \\ \mathcal{V}_f(\epsilon) = C_F\left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + 5 - \frac{\pi^2}{2}\right] \end{cases}$$

- Complete agreement with I terms for $pp \to t\bar{t} + 1\text{jet}$

S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C59 (2009) 625 and Phys.Rev.Lett.98(2007)262002

$$\sum_i \mathrm{I}_i = C_{-2}\frac{1}{\epsilon^2} + C_{-1}\frac{1}{\epsilon} + C_0$$

	c_{-2}	c_{-1}	c_0
		$gg \to t\bar{t}g$	
AutoDipole	$2.49467966948003 \cdot 10^{-4}$	$3.68989776683705 \cdot 10^{-4}$	$-4.05387364353899 \cdot 10^{-4}$
Version 2	$2.49467966948004 \cdot 10^{-4}$	$3.68989776683706 \cdot 10^{-4}$	$-4.05387364353900 \cdot 10^{-4}$
		$u\bar{u} \to t\bar{t}g$	
AutoDipole	$1.38499897972387 \cdot 10^{-5}$	$2.88738914389178 \cdot 10^{-5}$	$-1.56576469322102 \cdot 10^{-5}$
Version 2	$1.38499897972387 \cdot 10^{-5}$	$2.88738914389178 \cdot 10^{-5}$	$-1.56576469322102 \cdot 10^{-5}$
		$ug \to t\bar{t}u$	
AutoDipole	$3.84580760674706 \cdot 10^{-6}$	$7.73777480040817 \cdot 10^{-6}$	$-5.19929995897616 \cdot 10^{-6}$
Version 2	$3.84580760674706 \cdot 10^{-6}$	$7.73777480040817 \cdot 10^{-6}$	$-5.19929995897615 \cdot 10^{-6}$
		$g\bar{u} \to t\bar{t}\bar{u}$	
AutoDipole	$6.22738241305372 \cdot 10^{-5}$	$6.81530745255038 \cdot 10^{-5}$	$-1.52377227863896 \cdot 10^{-4}$
Version 2	$6.22738241305372 \cdot 10^{-5}$	$6.81530745255037 \cdot 10^{-5}$	$-1.52377227863896 \cdot 10^{-4}$

4. Outlook

- Summary

- Dipole subtraction : General and practical procedure in NLO QCD
- AutoDipole : Automated dipole subtraction
  - Mathematica code and an interface with MadGraph
  - D term: Version 1.0beta publicly available at the site of CAPP 09
  - I term: Completed
  - P and K terms: will be completed soon

- Plan
  - The complete package is publicly available soon
  - Compute new NLO QCD predictions of important background at LHC