Alternative dipole subtraction scheme using Nagy Soper dipoles

Tania Robens
in collaboration with
C. Chung, M. Krämer, Z. Nagy, D. Soper

RWTH Aachen University

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NLO corrections: general structure

Masterformula

for m particles in the final state

$$\begin{aligned}
\sigma_{\text{NLO,tot}} &= \sigma_{\text{LO}} + \sigma_{\text{NLO}}, \\
\sigma_{\text{LO}} &= \int d\Gamma_m \, |\mathcal{M}^{(m)}_{\text{Born}}|^2(s) && \text{leading order contribution} \\
\sigma_{\text{NLO}} &= \sigma_{\text{real}} + \sigma_{\text{virt}}, \\
\sigma_{\text{real}} &= \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2 && \text{real emission} \\
\sigma_{\text{virt}} &= \int d\Gamma_m \, 2\, Re(\mathcal{M}^{(m)}_{\text{Born}} \, (\mathcal{M}^{(m)}_{\text{virt}})^*) && \text{virtual contribution}
\end{aligned}$$

with $d\Gamma$: phase space integral, $\mathcal{M}$ matrix elements
(here: flux factors etc implicit)

Infrared divergencies in NLO corrections

- source of infrared divergence: integration over phase space of emitted massless particles in real and virtual contribution (poles cancel in $\sigma_{\text{real}} + \sigma_{\text{virt}}$)
- appear in matrix elements as terms $\frac{1}{p_i p_j} = \frac{1}{E_i E_j (1 - \cos\theta_{ij})}$
  $E_j \rightarrow 0$: soft divergence, $\cos\theta_{ij} \rightarrow 1$: collinear divergence
- poles arise from **integration** of phase space of p_j
- eg in QCD $\tilde{p}_i \rightarrow p_i + p_j$ (omitted color factors etc)

$$q \rightarrow q\,g \;:\propto\; \frac{1}{\varepsilon^2} + \frac{3}{2\,\varepsilon},\quad g \rightarrow q\,\bar{q} \;:\propto\; -\frac{1}{3\varepsilon}$$

- important: **this behaviour is the same for all processes**

Dipole subtraction: general idea

- know that pole structure always the same
- matrix element level: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j)\,|\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \qquad (1)$$

- D_{ij}: **dipoles**, contain complete singularity structure
- also means that

$$\int d\Gamma_{m+1} \left(|\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \text{finite}$$

- **general idea of dipole subtraction:** make use of (1), shift singular parts from $m+1$ to m particle phase space

$\Rightarrow$ **need to have a good (analytical) parametrization of the singularity structure**

Dipole subtraction for total cross sections

Master formula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C \right),$$

⇒ effectively added "0"; both integrals finite

$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \left\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 + \int_0^1 dx\, (\mathbf{K}(x) + \mathbf{P}(x;\mu_F))\, |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \right\}$$

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Ingredient for subtraction schemes: momentum matching

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \widetilde{F}_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2$$

- addition and subtraction takes place in different phase spaces

$$p^{(m)}_{\tilde{a}} = F\left(p^{(m+1)}_a, p^{(m+1)}_b, \ldots.\right)$$

This function is highly scheme dependent !!!

requirement: keep total energy/ momentum conserved:

$$\sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a$$

(sum over outgoing particles only)

Nagy Soper subtraction scheme

- many different subtraction schemes are around (best known: Catani, Seymour, 1996)
- all schemes: poles have to be the same; finite parts can differ

Main motivation for new scheme

- basic idea: can use the splitting functions in the parton shower as dipole subtraction terms
  $\Rightarrow$ have same behaviour in singular limits
- "turn around" of idea suggested by Nagy, Soper (hep-ph/0503053): use Catani Seymour Dipoles for shower algorithm
- introduce new matching between m and $m+1$ phase spaces

$\Rightarrow$ leads to a much smaller number of subtraction terms
especially important for large number of external particles

$\Rightarrow$ same dipoles in shower and subtraction scheme: facilitates matching with NLO calculations

Difference 1: Shifting momenta

- matching between m and $m+1$ particle spaces requires reshuffling of momenta
- for
$$p_{\text{mother}}^{(m)} = p_{\text{daughter, 1}}^{(m+1)} + p_{\text{daughter,2}}^{(m+1)},$$
not all particles can be onshell simultaneously

$\Rightarrow$ need additional spectators to take over additional momenta

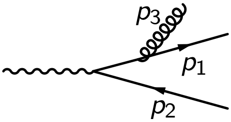
- Catani Seymour: define emitter-spectator pair, momentum goes to 1 additional particle only

$\Rightarrow$ quite easy integrations; however, for increasing number of particles, huge number of transformations necessary

- Nagy Soper:
shift momenta to **all** non-emitting external particles
- number of transformations = number of emitters
- leads to more complicated integrals during framework setup
- in general: # of transformations: CS $\sim N_{\text{jets}}^3/2$, NS$\sim N_{\text{jets}}^2/2$

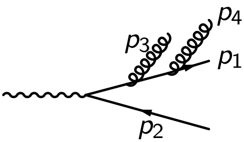
Shifting momenta: Example (1)

$$\gamma^* \longrightarrow q(p_1)\bar{q}(p_2)\, g(p_3) \text{ (@ NLO)}$$



part of Born contribution

real gluon emissions for this diagramm:

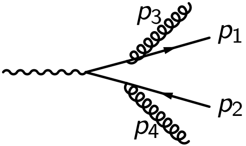


CS: 1 momentum shift/ spectator

p_2, p_3: 2 transformations

NS: 1 total transformation

Shifting momenta: Example (2)



CS: 1 momentum shift/ spectator

p_1, p_3: 2 transformations

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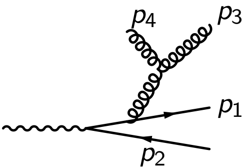


CS: 1 momentum shift/ spectator

p_1, p_2: 2 transformations

NS: 1 total transformation

$\Rightarrow$ from simple counting:

12 transformations using CS vs 6 using NS dipoles !!

of course many more contributions (eg $g \longrightarrow q\,\bar{q}$, other Born terms, ...)

Difference 2: Matching with parton showers

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- avoid double counting

$$-\int_{m+1} d\sigma^{\mathrm{PS}}|_{m+1} + \int_{m+1} d\sigma^{\mathrm{PS}}|_{m}$$

details eg in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

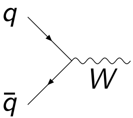
- important: have new terms in $m+1$ phase space

$$\int_{m+1} \left(d\sigma^{R} \underbrace{- d\sigma^{A} + d\sigma^{PS}|_{m}}_{=0} - d\sigma^{PS}|_{m+1} \right)$$

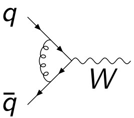
- same splitting functions: second and third term cancel analytically !!

$\Rightarrow$ improves numerical efficiency

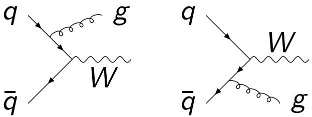
Single W production (slide by C. Chung)



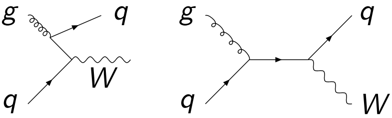
Tree level: $q\bar{q} \rightarrow W$



Virtual corrections: $q\bar{q} \rightarrow W$



Real corrections: $q\bar{q} \rightarrow Wg$



$gq \rightarrow Wq$ (+ 2 more diagrams)

$$\frac{1}{4}\frac{1}{9}|\mathcal{M}_B|^2 = \frac{g^2}{12}|V_{qq'}|^2 M_W^2, \quad \frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_R|^2 = \frac{8g^2\pi\alpha_s}{9}|V_{qq'}|^2 \frac{\hat{t}^2+\hat{u}^2+2M_W^2\hat{s}}{\hat{t}\hat{u}}$$

$$|\mathcal{M}_V|^2 = |\mathcal{M}_B|^2 \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right\}$$

Subtraction terms à la Nagy Soper

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \frac{8}{9}\,\pi\,\alpha_s\,g^2\left(\frac{t^2+u^2+2\,s\,p_3^2}{t\,u}\right) = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}}$$

- 1 particle phase space (virtual contribution)

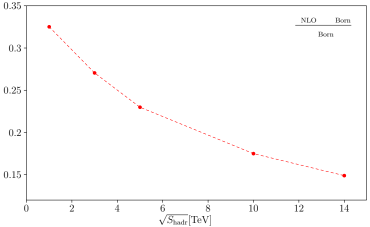
$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\mathbf{K}^a(xp_a) = \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\Big[-(1-x)\ln x + 2(1-x)\ln(1-x) + 4x\left(\frac{\ln 1-x}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+\ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right)\Big]$$

$$\mathbf{P}(x,\mu_F^2) = \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\left(\frac{1+x^2}{1-x}\right)_+\ln\left(\frac{4\pi\mu^2}{\mu_F^2}\right)$$

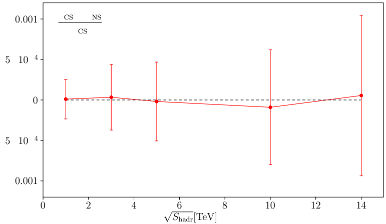
Numerical results for single W (slide by C. Chung)

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$



$\frac{\sigma_{NLO}-\sigma_{LO}}{\sigma_{LO}}$ as a function of $\sqrt{S_{\text{hadr}}}$

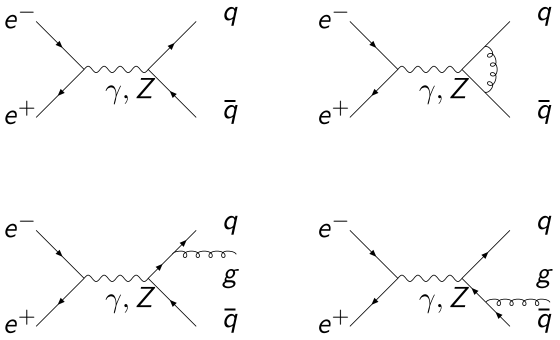
corrections up to 30%



relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$

agree on the sub-permill level ✓

Applications: $e^+e^- \longrightarrow 2\,\text{jets}$ (1) (slide by C.Chung)



Tree level diagram:
$e^+e^- \to q(p_1) + \bar{q}(p_2)$

Virtual corrections:
$e^+e^- \to q(p_1) + \bar{q}(p_2)$

Real corrections:
$e^+e^- \to$
$q(p_1) + \bar{q}(p_2) + g(p_3)$

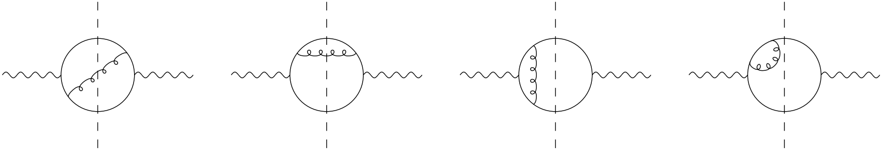
The matrix element for NLO real emission (three particle ps):

$$\mid \mathcal{M}_3(p_1, p_2, p_3) \mid^2 = \ C_F \frac{8\pi\alpha_s}{Q^2} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \mid \mathcal{M}_2 \mid^2, \ x_i = \frac{2 p_i \cdot Q}{Q^2}$$

($\mathcal{M}_2$, $\mathcal{M}_3$ averaged over angles)

soft/ collinear singularities from $x_i \to 1$

Applications: $e^+e^- \longrightarrow 2\,\text{jets}$ (2) (slide by C. Chung)



2 dipole contributions $\mathcal{D}_1$ and $\mathcal{D}_2$ (in 3 particle ps):

$$\mathcal{D}_1 = v^2_{qqg} - v^2_{\text{soft}} = \left(v^2_{qqg} - v^2_{\text{eik}}\right) + \left(v^2_{\text{eik}} - v^2_{\text{soft}}\right)$$

$$= \frac{4}{\hat{Q}^2}\left\{\left(\frac{1}{x_2}\right)\left[2\left(\frac{x_1}{2-x_1-x_2} - \frac{1-x_2}{(2-x_1-x_2)^2}\right) + \frac{1-x_1}{1-x_2}\right] + 2\left(\frac{x_1+x_2-1}{1-x_2}\right)\frac{x_1}{(1-x_1)x_1+(1-x_2)x_2}\right\}$$

Integration over dipole

$$2\left(\frac{4\pi\alpha_s}{2}\right)\mu^{2\epsilon}C_F\int d\zeta_p\mathcal{D}_1 = \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-2+\frac{\pi^2}{3}\right)$$

$$\sigma^{NLO} = \sigma^{NLO\,\{2\}} + \sigma^{NLO\,\{3\}} = \frac{3}{4}\frac{\alpha_s}{\pi}C_F\sigma^{LO} \quad (\checkmark)$$

Status quo (instead of Summary)

- goal: establish NS dipole formalism
- all integrals are done ✓
- need to countercheck a) singularities, b) finite terms
- a) almost completely done
  (missing: processes w more than 2 partons in the final state)
- b) almost done
  (missing: as a), + initial state final state interference terms)

Checked processes

- single W at hadron colliders
- Dijet production at lepton colliders
- $p\bar{p} \rightarrow H$ and $H \rightarrow g\,g$
- deep inelastic scattering:
  singularity cancellation checked, rest underway

Outlook

Outlook

- continue checks by application to simple processes for unchecked splitting functions
  (initial final interference, $m > 2$ in final state)
- implement on matrix element level
- match with parton shower (Z. Nagy; underway)
- apply in (new) higher order calculations
- .... (more to come)

! Thanks for listening !

Appendix

Processes at hadron colliders: general

- hadron colliders (as Tevatron, LHC) collide **hadrons**
- QCD: talks about **partons**
- transition: parton distribution functions (PDFs) $f_l(x, \mu_F)$; $l = q, \bar{q}, g$ flavour, x momentum fraction, (μ_F factorization scale)

masterformula

$$\sigma_{\text{hadr}}(p\,\bar{p} \to X) = \sum_{l_1, l_2} \int dx_1 \int dx_2 \, f_{l_1}(x_1) f_{l_2}(x_2) \, \sigma_{\text{part}}(x_1, x_2;\, l_1 l_2 \to X)$$

- perturbative, nonperturbative part

Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij},\ \widetilde{F}_{sing} = \int d\Gamma_1\, D_{ij},\ d\Gamma_1 \propto d^4p_j\,\delta(p_j^2)$$

$$\Longrightarrow \widetilde{F}_{sing} \propto \int d^4p_j\,\delta(p_j^2)\,D_{ij}$$

- 3 free variables (in D dimensions: $D-1$)

  !! need to be written in terms of m particle variables !!

- now all ingredients:

  total energy momentum conservation, onshellness of external particles, need for integration variables

Maximal number of transformations

emitter, spectator	CS	NS
fin,fin	$N'\,(N'-1)\,(N'-2)/2$	$N'\,(N'-1)/2$
fin,ini	$N'\,(N'-1)$	–
ini,fin	$2\,(N'-1)\,N'$	$2\,N'$
ini,ini	$2\,N'$	–
total	$N'^2(N'+3)/2 = (N+1)^2(N+4)/2$	$N'(N'+3)/2 = (N+1)(N+4)/2$

Table: Maximal number of transformations needed for N particles in the born final state ($N' = N+1$ in the real radiation contribution) using Catani Seymour or Nagy Soper prescriptions. Formula is exact for processes where all partons are gluons; for quarks, number can actually be smaller. Note: number of transformations is not equal to number of dipoles.

Dipole subtraction: Real master formula

Real Masterformula $(s = (p_a + p_b)^2)$

$$\begin{aligned}
\sigma(s) \;=\; & \int_m d\Phi^{(m)}(s) \frac{1}{n_c(a) n_c(b)} \, |\mathcal{M}^{(m)}|^2(s) F_J^{(m)} \\
& + \int d\Phi^{(m+1)}(s) \left\{ \frac{1}{n_c(a) n_c(b)} \, |\mathcal{M}^{(m+1)}|^2(s)) F_J^{(m+1)} \; - \sum_{\text{dipoles}} \left(\mathcal{D} \cdot F_J^{(m)}\right) \right\} \\
& + \int d\Phi^{(m)}(s) \left\{ \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m)}|^2_{1\,\text{loop}}(p_a, p_b) \; + \; \mathbf{I}(\varepsilon) |\mathcal{M}^{(m)}|^2(s) \right\}_{\varepsilon=0} F_J^{(m)} \\
& + \left\{ \int dx_a \, dx_b \delta(x - x_a) \, \delta(x_b - 1) \int d\Phi^{(m)}(x_a p_a, x_b p_b) |\mathcal{M}^{(m)}|^2 (x_a p_a, x_b p_b) \right. \\
& \left. \times \left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(x_a p_a, x_b p_b, x; \mu_F^2) \right) \right\} + (a \leftrightarrow b)
\end{aligned}$$

where all colour/ phase space factors have been accounted for

Integrated Dipoles in more details: I, K, P (1)

$m+1$ phase space: in principle easy

$$\int d\Gamma_{m+1} \left(|\mathcal{M}_{\text{real}}|^2 - \sum D \right), \quad \text{finite}$$

m particle phase space: more complicated

need integration variables (emission from p_1):

$$x = 1 - \frac{p_4(p_1 + p_2)}{p_1 p_2} \text{ softness}, \quad \tilde{v} = \frac{p_1 p_4}{p_1 p_2} \text{ collinearity}$$

Integrated Dipoles in more details: I, K, P (2)

- in principle, obtain $\int d\Gamma_1 \, D \,=\, \int_0^1 dx \, \left(\mathbf{I}(\varepsilon) + \tilde{\mathbf{K}}(x,\varepsilon)\right)$
- $\mathbf{I}(\varepsilon) \propto \delta(1-x)$: corresponds to loop part
- $\tilde{\mathbf{K}}(x,\varepsilon)$ contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^{\epsilon} P^{qq}(x)$$

  depends on factorization scale μ_F ($P^{qq}(x)$ splitting function)
- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted
- for $q\,g \rightarrow W\,q$ like processes, only singularity which appears

$q \longrightarrow q\, g$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} \;=\; -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,(s+t+u)}{t\,(t+u)} + (1-\varepsilon)\,\frac{t+u}{t}\right)$$

- matching ($\tilde{p}_2 \;=\; p_2$)

$$\tilde{p}_1 \;=\; x\,p_1, \quad x \;=\; 1 - \frac{p_4\,(p_1+p_2)}{(p_1\,p_2)}$$

$$\tilde{p}_k^{\mu} \;=\; \Lambda^{\mu}{}_{\nu}\,p_k^{\nu}, \quad (k\text{: final state particles})$$

$$\Lambda^{\mu\nu} \;=\; -g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^{\mu}(K+\widetilde{K})^{\nu}}{(K+\widetilde{K})^2} + \frac{2\,K^{\mu}\,\widetilde{K}^{\nu}}{K^2}$$

$$K = p_1 + p_2 - p_4, \quad \widetilde{K} = \tilde{p}_1 + p_2$$

$q \longrightarrow q\, g$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v = \frac{p_1 p_4}{p_1 p_2},\ x = 1 - \frac{p_4\,(p_1 + p_2)}{(p_1 p_2)}$$

- in p_1, p_2 cm system: $E_4 \longrightarrow 0 \Rightarrow x \longrightarrow 1$ (softness)
  $\cos\theta_{14} \longrightarrow 1 \Rightarrow v \longrightarrow 0$ (collinearity)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\,\pi\,\alpha_s\,C_F}{v\,x\,s}\left(\frac{1+x^2}{1-x} - \varepsilon(1-x)\right)$$

- integration measure

$$[dp_j] = \frac{(2\,p_1 p_2)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dv\,dx\,(1-x)^{-2\,\varepsilon}\left[\frac{v}{1-x}\left(1-\frac{v}{1-x}\right)\right]^{-\varepsilon}$$

where $v \leq 1 - x$ and all integrals between 0 and 1

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\varepsilon} \int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi} \frac{1}{\Gamma(1-\varepsilon)}\, C_F \left(\frac{2\mu^2\pi}{p_1 p_2}\right)^{\varepsilon}$$

$$\times \int_0^1 dx \left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon) \underbrace{-\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{I}(\epsilon) = \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+$$

$$P^{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ \quad \text{regularized splitting function}$$

$q \longrightarrow q\, g$ for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,u\,(s+t+u)}{t\,(t^2+u^2)} + (1-\varepsilon)\,\frac{u}{t}\right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure: as Catani Seymour($v \leftrightarrow y$)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s\,C_F}{x\,s} \times \left(\frac{1-x-y}{y}(1-\varepsilon) + \frac{2\,x}{y\,(1-x)} - \frac{2\,x\,[2\,y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]}\right)$$

$q \longrightarrow q\, g$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon} \int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi} \frac{1}{\Gamma(1-\varepsilon)}\, C_F \left(\frac{2\,\mu^2\pi}{p_1 p_2}\right)^{\varepsilon} \times \int_0^1 dx \left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon) \underbrace{-\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{K}(x) = (1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+ -(1-x)$$

- equivalence of dipoles schemes checked analytically

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (1)

- $g(\tilde{p}_i) \;\longrightarrow\; q(p_i) + \bar{q}(p_j)$,
  spectator: any other final state parton, p_k
- Dipole (in terms of integration variables):

$$D^{ij,k}_{\text{NS, CS}} \propto \underbrace{\frac{1}{y}}_{\text{sing}} \left[1 - \frac{z\,(1-z)}{1-\varepsilon}\right]$$

- NS definitions

$$y_{\text{NS}} = \frac{p_i\,p_j}{(p_i+p_j)Q - p_i\,p_j},\quad z_{\text{NS}} = \frac{p_j\,\tilde{n}}{p_i\,\tilde{n} + p_j\,\tilde{n}}$$

$$\tilde{n} = \frac{1+y+\lambda}{2\lambda}Q - \frac{a}{\lambda}\left(p_i+p_j\right),\ \lambda = \sqrt{(1+y)^2 - 4\,a\,y},\ a = \frac{Q^2}{(p_i+p_j)Q - p_i\,p_j}$$

- CS definitions:

$$y_{CS} = \frac{p_i\,p_j}{p_i\,p_j + p_i\,p_k + p_j\,p_k},\quad z_{CS} = \frac{p_i\,p_k}{p_i\,p_k + p_j\,p_k}$$

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (2)

- CS matching (all other final state particles untouched)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k, \;\; \tilde{p}_k^\mu = \frac{1}{1-y} p_k$$

- NS matching

$$\tilde{p}_i = \frac{1}{\lambda}(p_i + p_j) - \frac{1-\lambda+y}{2\,\lambda\,a} Q, \;\; \tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu \quad \textbf{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu (K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu \widetilde{K}^\nu}{K^2}, \; K = Q - p_i - p_j, \;\; \widetilde{K} = Q - \tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{CS} = \frac{(2\,\tilde{p}_i \tilde{p}_k)^{1-\varepsilon}}{16\,\pi^2} \frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}} dz\,dy\,(1-y)^{1-2\,\varepsilon} y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon},$$

$$[dp_j]_{NS} = \frac{(2\,\tilde{p}_i\,Q)^{1-\varepsilon}}{16\,\pi^2} \frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}} dz\,dy\,\lambda^{1-2\,\varepsilon} y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon}$$

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (3)

- result CS

$$\mu^{2\varepsilon}\int [dp_j]\, D^{ij,k} \;=\; \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,T_R\,\left(\frac{2\mu^2\pi}{\tilde{p}_i\tilde{p}_k}\right)^{\varepsilon}\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}\right]$$

- result NS

$$\mu^{2\varepsilon}\int [dp_j]\, D^{ij} \;=\; T_R\,\frac{\alpha_s}{2\,\pi}\,\frac{\alpha_s}{\Gamma(1-\varepsilon)}\left(\frac{2\,\pi\,\mu^2}{p_{\tilde{i}}Q}\right)^{\varepsilon}$$

$$\times\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}+\frac{2}{3}\left[(a-1)\,\ln(a-1)-a\,\ln\,a\right]\right],$$

- for $a=1$, reduces completely to Catani Seymour result
- (reason: $a = 1$ implies only 2 particles in the final state, $\tilde{n} \longrightarrow p_k$, $\Rightarrow$ complete equivalence)
- tradeoff: all final state particles get additional momenta: integral more complicated, but fewer transformations necessary