

Leptogenesis in the Kadanoff-Baym formalism: vertex contribution to the CP -violating parameter

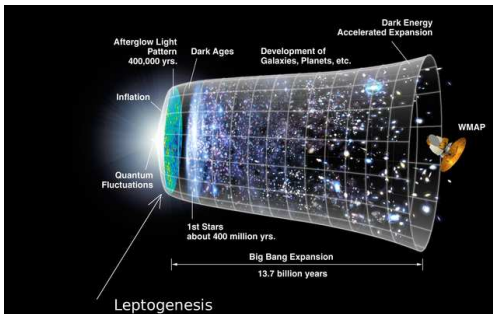
A. Kartavtsev

in collaboration with

M. Garny, A. Hohenegger and M. Lindner

MPI für Kernphysik, Heidelberg

Baryogenesis via leptogenesis

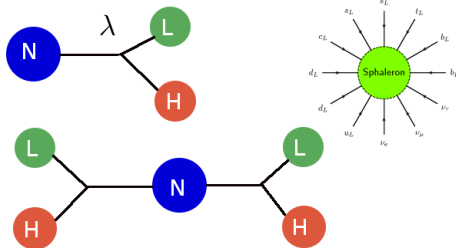


The universe is baryonically asymmetric:

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = 6.1_{-0.2}^{+0.3} \times 10^{-10}$$

Baryogenesis via leptogenesis:

- The Majorana mass term violates lepton number;
- Complex Yukawa couplings λ induce CP-violation;
- Expansion of the universe provides deviation from equilibrium;
- Sphalerons convert the lepton asymmetry to the baryon asymmetry.



Canonical approach

The Boltzmann equation for the lepton (baryon) distribution function:

$$\begin{aligned}
 p^\alpha \mathcal{D}_\alpha f_L &= \frac{1}{2} \int d\Pi_\alpha d\Pi_\beta \dots d\Pi_i d\Pi_j \dots \\
 &\times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \dots - p_i - p_j) \\
 &\times [|\mathcal{M}|_{i+j+\dots \rightarrow L+\alpha+\beta\dots}^2 f_i f_j \dots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \\
 &- f_L f_\alpha f_\beta \dots (1 \pm f_i)(1 \pm f_j) \dots |\mathcal{M}|_{L+\alpha+\beta\dots \rightarrow i+j+\dots}^2].
 \end{aligned}$$

Pros:

- Easy to derive and solve numerically.

Cons:

- Relies on the quasiparticle picture;
- Matrix elements \mathcal{M} are calculated in *vacuum*.
- Double-counting problem;

Canonical approach

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 \end{aligned}$$

Cons:

- Double-counting problem:

$$\begin{aligned}
 & \begin{array}{c} f_L \\ \diagdown \\ \text{---} \circ \text{---} \\ \diagup \\ f_H \end{array} \text{---} (1 \pm f_N) \quad - \quad \begin{array}{c} (1 \pm f_L) \\ \diagup \\ \text{---} \circ \text{---} \\ \diagdown \\ (1 \pm f_H) \end{array} \\
 & |\mathcal{M}|^2 = |\mathcal{M}_0|^2(1 - \epsilon) \qquad \qquad \qquad |\mathcal{M}|^2 = |\mathcal{M}_0|^2(1 + \epsilon)
 \end{aligned}$$

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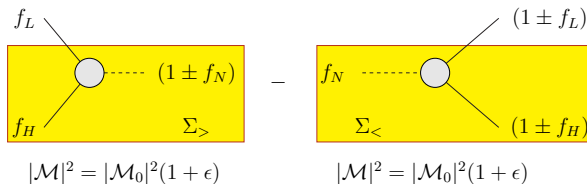
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From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

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 &\times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \dots - p_i - p_j) \quad \Sigma_{<}, \text{ gain term} \\
 &\times [|\mathcal{M}|_{i+j+\dots \rightarrow L+\alpha+\beta \dots}^2 f_i f_j \dots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \\
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 &\quad \Sigma_{>}, \text{ loss term}
 \end{aligned}$$

Gain and loss terms:



From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

$$p^\alpha \mathcal{D}_\alpha f_L = \frac{1}{2} [\Sigma_<(X, p)(1 \pm f_L) - f_L \Sigma_>(X, p)].$$

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Now we change the notation.

- Spectral function:

$$D_\rho(X, p) = (2\pi) \text{sign}(p_0) \delta(p^2 - m^2), \quad p^\alpha \mathcal{D}_\alpha D_\rho(X, p) = 0.$$

- Statistical propagator:

$$D_F(X, p) = \left[\frac{1}{2} \pm f_L(X, p) \right] D_\rho(X, p).$$

- For further convenience:

$$D_{\gtrless}(X, p) \equiv D_F(X, p) \pm \frac{1}{2} D_\rho(X, p).$$

From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

$$p^\alpha \mathcal{D}_\alpha D_F(X, p) = \frac{1}{2} [\Sigma_{<}(X, p) D_{>}(X, p) - D_{<}(X, p) \Sigma_{>}(X, p)],$$

$$p^\alpha \mathcal{D}_\alpha D_\rho(X, p) = 0.$$

What have we gained ?

- So far, only a change of notation;
- However, coincide with the equations that can be derived from the Kadanoff-Baym equations.
- The Kadanoff-Baym gain and loss terms Σ_{\gtrless} differ from the Boltzmann gain and loss terms.

Kadanoff-Baym equations

Kadanoff-Baym equations – a coupled system of integro-differential equations for the spectral function and statistical propagator:

$$\begin{aligned}
 [\square_x + m^2(x)]D_F(x, y) &= \int_0^{y^0} \mathcal{D}^4 z \Sigma_F(x, z) D_\rho(z, y) \\
 &\quad - \int_0^{x^0} \mathcal{D}^4 z \Sigma_\rho(x, z) D_F(z, y), \\
 [\square_x + m^2(x)]D_\rho(x, y) &= \int_{x^0}^{y^0} \mathcal{D}^4 z \Sigma_\rho(x, z) D_\rho(z, y).
 \end{aligned}$$

Pros:

- Do not rely on the concept of quasiparticles;
- Take the medium effects into account;
- Free of the double counting problem.

Cons:

- Complexity.

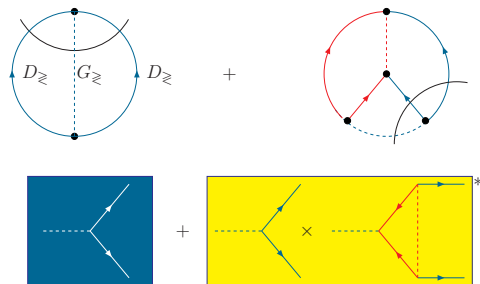
Toy model of leptogenesis

Lagrangian of the model reads:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial^\mu \psi_i \partial_\mu \psi_i - \frac{1}{2} M_i^2 \psi_i \psi_i \\ & + \partial^\mu \bar{b} \partial_\mu b - m^2 \bar{b} b - \frac{\lambda}{2!2!} (\bar{b} b)^2 \\ & - \frac{g_i}{2!} \psi_i b b - \frac{g_i^*}{2!} \psi_i \bar{b} \bar{b}, \quad i = 1, 2. \end{aligned}$$

- Real scalar fields ψ_i imitate the heavy right-handed neutrinos;
- Complex scalar field b models the baryons;
- The baryon number is broken by the last two terms of \mathcal{L} ;
- Complex couplings g_i induce CP-violation;
- Deviation from equilibrium is due to the universe's expansion;
- The $\lambda(\bar{b}b)^2$ term models fast SM interactions.

Gain and loss terms



Prescription:

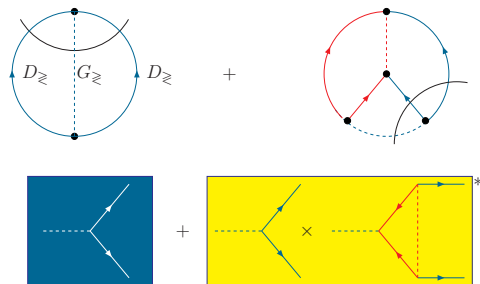
- Draw all 2PI diagrams and read off the effective action;
- Cut one line to obtain the self-energies;
- Cut the diagrams in two pieces to understand which processes it describes.

At one-loop level the decay self-energies read:

$$\Sigma_{\geq}(X, p) = - |g_i|^2 \int d\Pi_{p_1} d\Pi_{p_2} (2\pi)^4 \delta^g(p_1 - p_2 - p) \\ \times [1 + \epsilon_i(X, p_1, p_2)] \tilde{G}_{\geq}^{ii}(X, p_1) \tilde{D}_{\leq}(X, p_2).$$

- The loop correction is the same for both the gain and loss terms.

Gain and loss terms



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Conclusions:

At one-

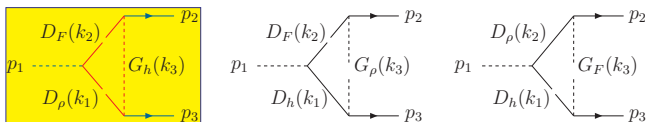
- No need for Real Intermediate State subtraction;

$$\Sigma_{\geq}(X, p) = - |g_i|^2 \int d\Pi_{p_1} d\Pi_{p_2} (2\pi)^4 \delta^g(p_1 - p_2 - p) \\ \times [1 + \epsilon_i(X, p_1, p_2)] \tilde{G}_{\geq}^{ii}(X, p_1) \tilde{D}_{\leq}(X, p_2).$$

- The loop correction is the same for both the gain and loss terms.

CP-violating parameter

Vertex contribution:



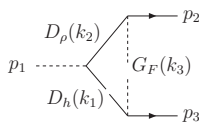
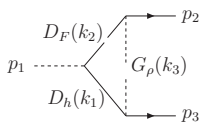
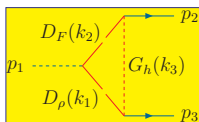
- One off-shell and two on-shell internal lines;
- Only one ‘thermal’ internal line;
- Only the diagram with two on-shell ‘baryons’ contributes;

Explicit form

$$\epsilon_i(p_1, p_2) = -\frac{1}{8\pi} \frac{|g_j|^2}{M_i^2} \text{Im} \left(\frac{g_i g_j^*}{g_i^* g_j} \right) \int \frac{d\Omega}{4\pi} \frac{1 + \bar{f}(E_{k_1}) + \bar{f}(E_{k_2})}{M_j^2/M_i^2 + \frac{1}{2}(1 + \cos \Theta)}$$

CP-violating parameter

Vertex contribution:



- One off-shell and two on-shell internal lines;

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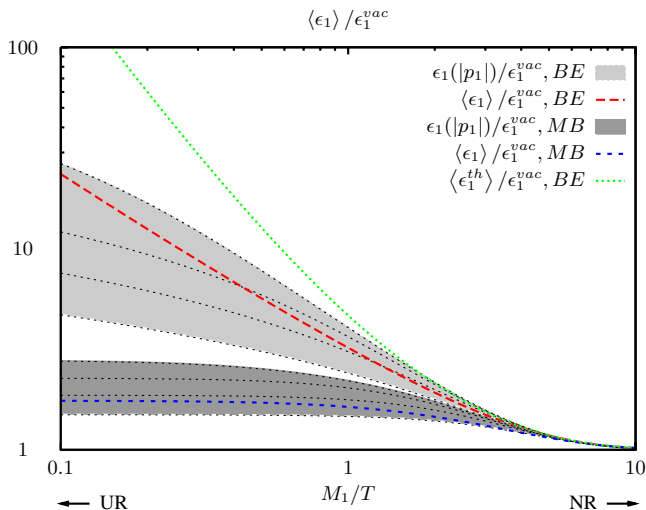
Conclusions:

- Or
 - No need for Real Intermediate State subtraction;
 - Our results differ from those of thermal field theory;

Explicit result

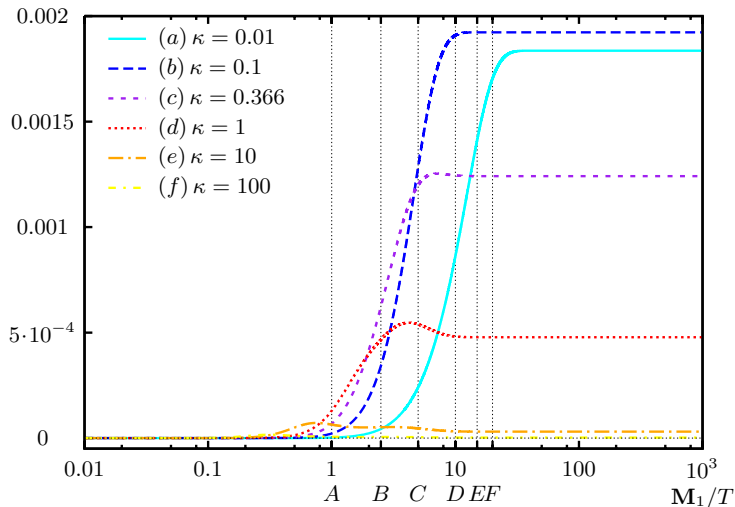
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Some numerical results



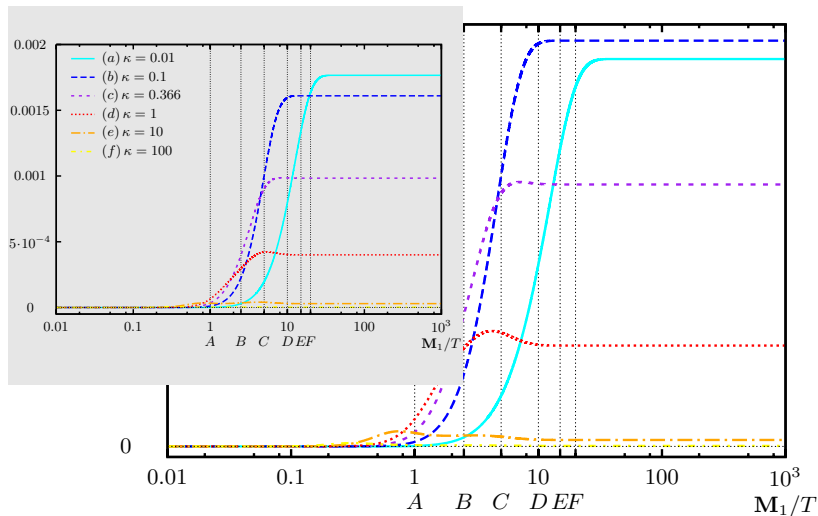
Temperature dependence of the CP-violating parameter.

Some numerical results



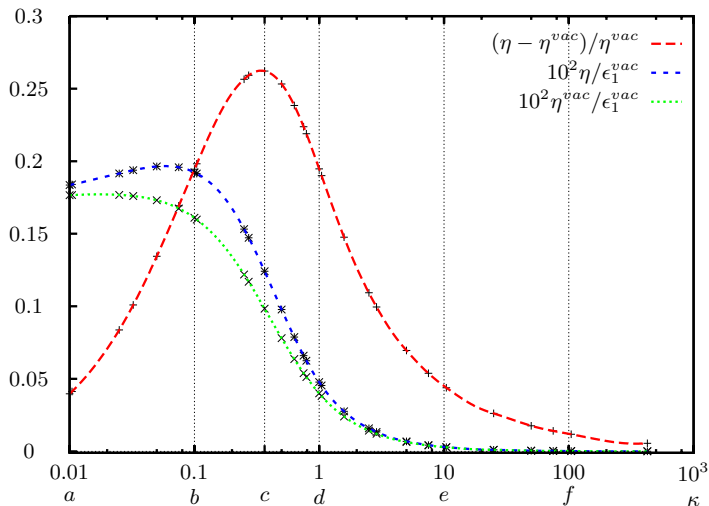
Asymmetry as a function of the inverse temperature.

Some numerical results



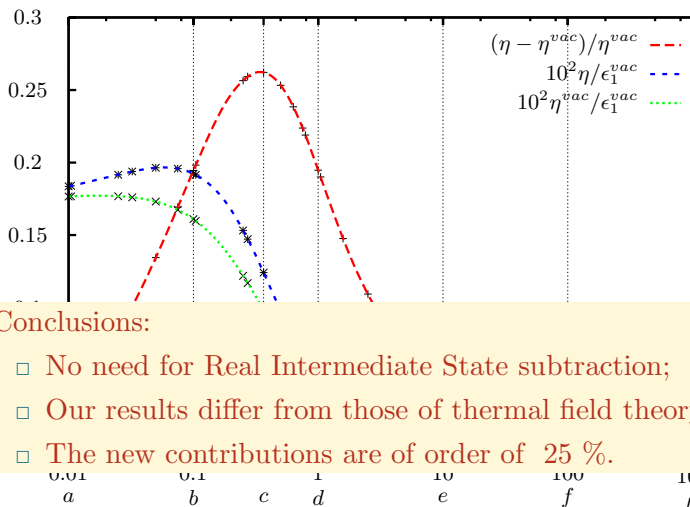
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Asymmetry as a function of $\kappa \equiv \Gamma/\mathcal{H}$

Some numerical results



Conclusions:

- No need for Real Intermediate State subtraction;
- Our results differ from those of thermal field theory;
- The new contributions are of order of 25 %.

Asymmetry as a function of $\kappa \equiv \Gamma/\mathcal{H}$

Conclusions

1 Kadanoff-Baym formalism:

Pros.

- Do not rely on the concept of quasiparticles;
- Take the medium effects into account;
- Free of the double counting problem.

Cons.

- Complexity.

2 To use or not to use ?

- The new contributions are of order of 25 %;
- No need for Real Intermediate State subtraction;
- Our results differ from those of thermal field theory.

Thank you for your attention